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H. F. NEWALL, Esq., M.A., F.R.S., PRESIDENT, in the Chair.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended :—

Harold A. H. Christie, B.A., Royal Observatory, Greenwich (proposed by Sir W. H. M. Christie);
Bertram Francis Eardley Keeling, The Observatory, Helwan, near Cairo, Egypt (proposed by H. H. Turner); and
Frederick Alexander Lindemann, Sidholme, Sidmouth, Devon (proposed by A. F. Lindemann).

Sixty-nine presents were announced as having been received since the last meeting, including, amongst others :—

Maurice Farman, Mesures d'étoiles doubles, presented by the author; Harvard Observatory Annals, vol. 52, part i, Eclipses of Jupiter's satellites; vol. 52, part 2, Second Catalogue of variable stars, presented by the Observatory; Heidelberg Astrophysical Observatory, Publications, vol. 2, Nos. 2-10, presented by the Observatory; Meudon Observatory, Annals, vol. 3, part i, presented by the Observatory; 20 charts of the Astrographic Chart of the heavens, presented by the Royal Observatory, Greenwich.

Note on Le Verrier's Tables of Saturn. By A. M. W. Downing, D.Sc., F.R.S.

My attention has been called to the rapidly-increasing discordance between the places of Saturn derived from Le Verrier's tables and those derived from the tables of Hill, as shown by the latest available data, *i.e.* the Ephemerides for 1909.

The following statement gives the corrections to Le Verrier's geocentric places, near the time of opposition in 1909, as given in the *Connaissance des Temps* for that year, deduced from their comparison with the corresponding places given in the *Nautical Almanac* :—

Day 1909.	R.A.	Dec.
	s	″
Oct. 2	−1·09	−6·8
10	−1·10	−6·9
18	−1·08	−6·8

The corresponding corrections to the heliocentric longitudes are :—

Day 1909.	Longitude
	″
Oct. 2	−16·0
10	−15·9
18	−15·9

It will be remembered that M. Gaillot has published "Tables rectifiées du mouvement de Saturne" in tome xxiv. of the *Annales de l'Observatoire de Paris*, and it is of interest to exhibit the resulting corrections to Le Verrier's heliocentric longitudes according to Hill, and according to Gaillot, at convenient epochs, as follows :—

Epoch.	Hill.	Gaillot.
	″	″
1894 March 6	− 3·8	...
1901 Jan. 1	− 3·5	− 4·9
1909 Oct. 10	−15·9	−16·2

The observed mean correction to Le Verrier's longitudes of Saturn, derived from the Greenwich Observations of 1900, is −4″, whilst the observed mean correction to Hill's longitudes, derived from the Greenwich Observations of the following year (the first year in which Hill's tables were used in the *Nautical Almanac*), is about half a second in the same sense; thus demonstrating the superior accuracy of Hill and Gaillot at the commencement of the century. But it is important to note the rapid increase in the discordance between Le Verrier and the other two authorities since that time.

Computation of Secular Perturbations. By R. T. A. Innes.

I.

It is well known that the Lagrangian equations for the variation of a planet's elements (the planet being considered of infinitesimal dimensions) are exact. It is in the integration of these equations that difficulties arise. To avoid these difficulties several assumptions are made, such as that the planets can be considered in pairs (*i.e.* the problem of three bodies) and do not react on each other; that the elements on the right-hand side of the equations are constant, and that the perturbative function can be developed in a converging series. When we are concerned with the mutual perturbations of the eight major planets, the above assumptions are sufficiently good; the errors introduced by the first two assumptions are eliminated later by taking into account the powers of the masses higher than the first, the third assumption is nearly justified by the smallness of the eccentricities and mutual inclinations of the eight major planets. In cases of difficulty, the Lagrangian equations are abandoned and the perturbations found by other processes; thus Hill found that the Lagrangian method as used by Le Verrier for the theory of the motion of Jupiter and Saturn was insufficient, so that in his theory he adopted the processes developed by Hansen.

Gauss showed that the secular part of the perturbations (to the first power of the masses) could be found without using a development of the perturbative function. The rationale of Gauss's method is as follows:—Let us imagine that the perturbative function is developed in a series as follows,

$$P_0 = P_1 + P_s \sin M + P_c \cos M + P_{2s} \sin 2M + \text{etc.}$$

where the coefficients P contain, besides certain quantities depending on the elements of the disturbed planet, the elements and position of the disturbing body. Multiplying each side by dM and integrating around the circumference gives

$$\frac{1}{2\pi}\int_0^{2\pi} P_0 dM = P_1$$

Then writing $P_1 = p_1 + p_s \sin M_1 + p_c \cos M_1 + \text{etc.}$, where the coefficients p depend only on the elements of both planets and do not contain the time explicitly, a second integration gives

$$\frac{1}{4\pi^2}\int_0^{2\pi}\int_0^{2\pi} P_0 dM dM_1 = p_1$$

It is on the quantity p_1 that the secular perturbations depend. In the general case it is impossible to find this quantity by algebraical expansions. Gauss proved that one of these integrations could be

reduced to elliptic integrals; the second has to be a mechanical integration. Gauss, as is well-known, gave a geometrical title to his paper and did not prepare his formulæ for numerical application. Bour and others have given further geometrical interpretations of Gauss's formulæ, which, interesting as they are, call for no attention here.

Since Dr. G. W. Hill published his first paper on Gauss's Method of Computing the Secular Perturbations of the Planets (Astron. Papers prepared for the use of the American Ephemeris, 1882; Collected Works, T. ii., 1906) many numerical applications of this method have been made. Hill did not leave the method in final shape, for no sooner had he finished his paper than he proceeded to give a substantial modification in an addendum by making use of Gauss's arithmetico-geometrical mean. Both of Hill's solutions require extensive tables of the elliptic integrals for their easy application. Hill provided these tables for his first solution, and Callandreau and the writer did the same for Hill's second method. Previously to Hill's work there had been but little use made of Gauss's method. We learn from Bode's *Astronomisches Jahrbuch*, 1819, p. 229, that before Gauss had actually published his memoir, Nicolai had computed the sec.-var. of the elements of the Earth's orbit by this method, but no details of the work were given. The late Professor J. C. Adams applied the method to the perturbation of the orbit of the November Meteors (see *M. N.*, 1867, Apl.; Collected Works, T. ii. pp. 194–200). It is well known that the method as expounded by Gauss requires the solution of a cubic equation, but Adams avoids the cubic by neglecting a portion of the terms factored by the square of the eccentricity of the disturbing body, and so reduces the equation to a quadratic. This is no doubt the modification which Adams stated greatly facilitates the application of Gauss's formulæ. As a practical step this modification was quite justified, but it takes away from the interest of the problem, and, as will appear immediately, is of no real advantage. Halphen, in 1886 (C.R., T. ciii.) announced that the problem could be completely solved without the labour of obtaining the roots of the cubic; he gave an exposition of his method in his *Fonctions Elliptiques*, 1888, T. ii. I was unable to follow Halphen's exposition, to which Tisserand's explanation added nothing (*Mec. Cel.*, T. i.), and it was only on obtaining by chance a copy of Dr Louis Arndt's paper on Halphen's solution (*Recherches . . . . Pert. Seculaires*, Neuchâtel, 1896) that the solution was grasped. The formulæ for numerical application given by Arndt assume that g_3 is always positive, so that his solution will only apply to the easier cases, although Halphen drew special attention to the case of g_3 being negative. Arndt has provided tables for the cases in which Legendre's modular angle θ will not exceed 45°. Hill has again revised his solution, and shows that by using Jacobi's nome q tables are practically unnecessary, but the roots of the cubic are still required (*Ast. Jour.*, No. 511, 1901).

As already stated, Gauss's method is rigorous to the first power

of the masses. It has been applied to most of the mutual actions of the major planets, but unnecessarily so in many cases, as the tables given by Newcomb and Le Verrier are ample and much simpler to use unless the ratio of the mean distances exceeds one-half; in other words, the small inclinations and eccentricities of the orbits of the major planets permit algebraical expansion, which only becomes long when this ratio exceeds one-half.

This method has its rôle in computing the first order secular perturbations of the periodical comets and minor planets. An interesting application to Eros has recently been made by Herr W. Dziewulski (*Säkulare Marsstörungen . . . des Eros*, Cracovie, 1906). He shows incidentally that this method gives better results in the case of Eros than the Lagrangian method, which takes into account all powers of the masses, but not of the inclinations and eccentricities. In fact, the Lagrangian approximation will in some cases not even give the correct signs, far less the correct figures. Tolerable approximations are only given in the cases of the planets with preponderating masses.

Mr C. J. Merfield, of Sydney, has recently announced the completion of a computation of the secular variation of Eros.

In this paper Hill's first exposition has been adopted as a working basis, and in the introduction of the elliptic functions Schwarz's notation, as developed in Padé's translation (*Formules et Propositions*, Paris, 1894), has been followed, except that the quantity h is replaced by Jacobi's letter q. No proofs of formulæ found in previous papers are given.

The aim throughout has been simplicity in numerical application. The final formulæ have been framed for that end, and have been submitted to numerical tests. The pure mathematician dismisses one with a series guaranteed to be absolutely convergent for certain values of the argument, but a trial often proves that the series, though it may be absolutely convergent, is arithmetically absolutely useless. It will appear that if an accuracy of six places of figures suffices, the use of series can be entirely evaded, a result by no means to be expected *a priori*.

The two main points in this paper are (1) the simplification of the formulæ by the introduction of the Weierstrassian cubic; (2) the new series for the computation of the periods of the elliptic functions,—series which are remarkably inert, nearly all the variation being thrown on to a simple trigonometrical factor of the series.

Thus the computer has the choice of two main methods in applying Gauss's principle,—(1) solving the cubic and using Jacobi's nome q (Hill, *Ast. Jour.*, No. 511); or (2) using the invariants of the cubic and two hypergeometrical series. Either of these methods is much shorter than the earlier solutions; which of the two is the better in actual practice cannot be answered offhand. It may be remarked that the former uses almost entirely trigonometrical logarithms, which the latter does not, and this alone is a considerable advantage.

2.

All the equations and formulæ wanted in practice are collected later on. Here it is unnecessary to write down the six Lagrangian equations; as a type we confine our attention to that for the eccentricity

$$\frac{de}{dt} = \cos\phi \frac{d\phi}{dt} = \frac{a^2 n \cos\phi}{1+m}\left[\sin v\,\mathrm{R} + (\cos v + \cos \mathrm{E})\mathrm{S}\right]$$

We have to find that part of the values of R, S, and W which is independent of the time (or of the positions of the planets in their orbits).

As already stated, two integrations are required, one of which will be done mechanically. The above equation is therefore written

$$\left[\frac{d\phi}{dt}\right]_{00} = \frac{m_1 n}{1+m}\mathrm{M_E}\left[\cos\phi \sin \mathrm{E}\frac{a}{r}\mathrm{R}_0 - \left(\frac{3}{2}e - 2\cos \mathrm{E} + \frac{e}{2}\cos 2\mathrm{E}\right)\frac{a}{r}\mathrm{S}_0\right]$$

The mechanical integration need only be made on the quantities

$\frac{a}{r}\mathrm{R}_0$, $\frac{a}{r}\mathrm{S}_0$, and $\frac{a}{r}\mathrm{W}_0$, where

$$\frac{a}{r}\mathrm{R}_0 = \mathrm{A}_0^{(c)} + \frac{1}{2}\mathrm{A}_1^{(c)}\cos \mathrm{E} + \frac{1}{2}\mathrm{A}_1^{(s)}\sin \mathrm{E} + \frac{1}{2}\mathrm{A}_2^{(c)}\cos 2\mathrm{E} + \text{etc.}$$

with similar equations for $\frac{a}{r}\mathrm{S}_0$ and $\frac{a}{r}\mathrm{W}_0$.

Thus the final equations take the form

$$\left[\frac{d\phi}{dt}\right]_{00} = \frac{m_1 n}{1+m}\left[\frac{1}{2}\mathrm{A}_1^{(s)}\cos\phi - \frac{3}{2}e\mathrm{B}_0^{(c)} + \mathrm{B}_1^{(c)} - \frac{e}{4}\mathrm{B}_2^{(c)}\right]$$

The addendum to Hill's first paper may be referred to for further matter concerning these equations.

We have

$$\frac{a}{r}\mathrm{R}_0 = \frac{1}{2\pi}\int_0^{2\pi} \frac{a^2}{r}\,\frac{xx_1 + yy_1 - r^2}{\Delta^3}(1 - e_1\cos \mathrm{E}_1)d\mathrm{E}_1$$

with corresponding expressions of the same form for $\frac{a}{r}\mathrm{S}_0$ and $\frac{a}{r}\mathrm{W}_0$.

Putting

$$\frac{a}{a_1} = \alpha \qquad r_0 = a(1 - e\cos \mathrm{E}) = \frac{r}{a_1}$$

$$\begin{aligned} k\cos(v+\mathrm{K}) &= \mathrm{A}_c & k_1\cos\phi_1\sin(v+\mathrm{K}_1) &= \mathrm{A}_s \\ -k\sin(v+\mathrm{K}) &= \mathrm{B}_c & k_1\cos\phi_1\cos(v+\mathrm{K}_1) &= \mathrm{B}_s \\ \sin\Pi_1 &= \mathrm{C}_c & \cos\phi_1\cos\Pi_1 &= \mathrm{C}_s \end{aligned}$$

$$\mathrm{A}_0 = 1 + r_0^2 + 2e_1 r_0 \mathrm{A}_c \qquad \text{Note, Hill's } \mathrm{A} = a_1^2 \mathrm{A}_0, \text{ etc.}$$

$$\mathrm{B}_0\cos\epsilon = r_0\mathrm{A}_c + e_1$$

$$\mathrm{B}_0\sin\epsilon = r_0\mathrm{A}_s$$

$$\mathrm{C}_0 = e_1^2.$$

$$\Delta_0^2 = \mathrm{A}_0 - 2\mathrm{B}_0\cos(\mathrm{E}_1 - \epsilon) + \mathrm{C}_0\cos^2\mathrm{E}_1$$

$$\text{Hill's } \Delta = a_1\Delta_0$$

then

$$\frac{a}{r}\mathrm{R}_0 = \frac{1}{2\pi}\int_0^{2\pi} \alpha^2 \frac{\mathrm{A}_c(\cos\mathrm{E}_1 - e_1) + \mathrm{A}_s\sin\mathrm{E}_1}{\Delta_0^3} - r_0(1 - e_1\cos\mathrm{E}_1)d\mathrm{E}_1$$

$$\frac{a}{r}\mathrm{S}_0 = \frac{1}{2\pi}\int_0^{2\pi} \alpha^2 \frac{\mathrm{B}_c(\cos\mathrm{E}_1 - e_1) + \mathrm{B}_s\sin\mathrm{E}_1}{\Delta_0^3}(1 - e_1\cos\mathrm{E}_1)d\mathrm{E}_1$$

$$\frac{a}{r}\mathrm{W}_0 = \frac{1}{2\pi}\int_0^{2\pi} a\sin\mathrm{J}\, r_0 \frac{\mathrm{C}_c(\cos\mathrm{E}_1 - e_1) + \mathrm{C}_s\sin\mathrm{E}_1}{\Delta_0^3}(1 - e_1\cos\mathrm{E}_1)d\mathrm{E}_1$$

If we remove the constant factors a^2, a^2, and $a\sin\mathrm{J}$ respectively, the numerators of the fractions on the right may be written as

$$f - ge_1 + [g(1+e_1^2) - fe_1]\cos\mathrm{E}_1 + h\sin\mathrm{E}_1 - he_1\sin\mathrm{E}_1\cos\mathrm{E}_1 - ge_1\cos^2\mathrm{E}_1,$$

where f, g, and h have the respective values

	$\frac{1}{a^2}\frac{a}{r_0}\mathrm{R}_0$	$\frac{1}{a^2}\frac{a}{r_0}\mathrm{S}_0$	$\frac{1}{a\sin\mathrm{J}}\frac{a}{r_0}\mathrm{W}_0 \times \frac{1}{r_0}$
f	$-r_0$	0	0
g	A_c	B_c	C_c
h	A_s	B_s	C_s

3.

Altering the variable from E_1 to T changes the equations for $\frac{1}{a^2}\frac{a}{r_0}R_0$, etc., to the form

$$\frac{1}{2\pi}\int_0^{2\pi}\frac{\Gamma_1+\Gamma_2\sin^2 T+\Gamma_3\cos^2 T}{\Delta_0^3}\,dT \qquad . \qquad . \quad (a)$$

wherein

$$\Delta_0^2 \quad \text{is now} \quad G_1-G_2\sin^2 T+G_3\cos^2 T$$

but we may now omit the subscript $_0$ as unnecessary.

The elliptic integrals, using the limits o to $\frac{\pi}{2}$, are

$$\int_0^{\frac{\pi}{2}}\frac{dT}{\Delta^3}, \qquad \int_0^{\frac{\pi}{2}}\frac{\sin^2 T dT}{\Delta^3} \quad \text{and} \quad \int_0^{\frac{\pi}{2}}\frac{\cos^2 T dT}{\Delta^3}$$

The G's are the roots of the equation

$$x^3-(A-C)x^2+(B^2-AC)x+CB^2\sin^2\epsilon=0$$

or $$x^3-k_1x^2+k_2x-k_3=0$$

Putting

$$e_1=G_1-\frac{k_1}{3}, \qquad e_2=G_2-\frac{k_1}{3}, \qquad e_3=-G_3-\frac{k_1}{3}$$

transforms

$$G_1-G_2\sin^2 T+G_3\cos^2 T \quad \text{into} \quad e_1-e_2\sin^2 T-e_3\cos^2 T$$

Introducing the Weierstrassian functions by means of

$$\sin^2 T=\frac{e_1-e_3}{e_2-e_3}\cdot\frac{s-e_2}{s-e_1} \qquad \left(\text{when } T=\begin{cases} o, & s=e_2 \\ \frac{\pi}{2}, & s=e_3 \end{cases}\right.$$

gives

$$\cos^2 T=\frac{e_2-e_1}{e_2-e_3}\cdot\frac{s-e_3}{s-e_1}$$

and

$$\frac{dT}{ds}=\frac{1}{2}\sqrt{\frac{(e_1-e_3)(e_2-e_1)}{(s-e_2)(s-e_3)}}\cdot\frac{1}{s-e_1}$$

$$\Delta^2=(e_1-e_2)(e_1-e_3)/(s-e_1)$$

$$\frac{dT}{\Delta^3}=\frac{1}{2}\frac{s-e_1}{(e_1-e_2)(e_1-e_3)}\cdot\frac{ds}{\sqrt{(s-e_1)(s-e_2)(s-e_3)}}$$

Putting

$$s = \mathfrak{p}u \quad \text{and therefore} \quad ds = \mathfrak{p}'u du$$

and noting that

$$\mathfrak{p}'u = 2\sqrt{(\mathfrak{p}u - e_1)(\mathfrak{p}u - e_2)(\mathfrak{p}u - e_3)}$$

we have

$$\frac{d\mathrm{T}}{\Delta^3} = \frac{\mathfrak{p}u - e_1}{(e_1 - e_2)(e_1 - e_3)} du$$

Having $\mathfrak{p}u = e_3$ when $u = \omega'$ and to e_2 when $u = \omega''$ or $\omega + \omega'$ and writing $\sqrt{G} = (e_1 - e_2)(e_1 - e_3)(e_2 - e_3)$ and noting that

$$-\mathfrak{p}u du = d\frac{\mathfrak{S}_1}{\mathfrak{S}} u$$

or

$$\int \mathfrak{p}u du = -\frac{\mathfrak{S}_1}{\mathfrak{S}} u$$

so that

$$\int_{\omega''}^{\omega'} \mathfrak{p}u du = \eta$$

we obtain

$$\int_0^{\frac{\pi}{2}} \frac{d\mathrm{T}}{\Delta^3} = \frac{e_2 - e_3}{\sqrt{G}}(e_1\omega + \eta)$$

$$\int_0^{\frac{\pi}{2}} \frac{\sin^2 \mathrm{T} d\mathrm{T}}{\Delta^3} = \frac{e_1 - e_3}{\sqrt{G}}(e_2\omega + \eta) \quad . \quad . \quad . \quad (b)$$

$$\int_0^{\frac{\pi}{2}} \frac{\cos^2 \mathrm{T} d\mathrm{T}}{\Delta^3} = \frac{e_2 - e_1}{\sqrt{G}}(e_3\omega + \eta)$$

The proofs of the above formulæ will be found in Padé's or Schwarz's paper already referred to. The italic G is not to be confounded with the three G roots.

The last set of formulæ correspond to Dr. Hill's $\frac{\mathrm{M}}{m^3}$, $\frac{\mathrm{M}}{m^3}\cos^2\kappa$ and $\frac{\mathrm{M}}{m^3}\sin^2\kappa$ in the *Amer. Journ. of Math.*, t. xxiii. pp. 321–2.

Should the cubic equation be solved, the numerical value of these formulæ may be found as follows:—

$$\frac{2}{\pi}\int_0^{\frac{\pi}{2}} \frac{d\mathrm{T}}{\Delta^3} = \sqrt{\frac{e_2 - e_3}{G}}\ \frac{1}{4\sqrt{q}}\ \frac{1 + 3^2q^2 + 5^2q^6 + 7^2q^{12} + \text{etc.}}{(1 + q^2 + q^6 + q^{12} + \text{etc.})^3}$$

$$\frac{2}{\pi}\int_0^{\frac{\pi}{2}} \frac{\sin^2 \mathrm{T} d\mathrm{T}}{\Delta^3} = \sqrt{\frac{e_1 - e_3}{G}}\ 8q\ \frac{1 + 2^2q^3 + 3^2q^8 + 4^2q^{15} + \text{etc.}}{(1 + 2q + 2q^4 + 2q^9 + \text{etc.})^3}$$

$$\frac{2}{\pi}\int_0^{\frac{\pi}{2}} \frac{\cos^2 \mathrm{T} d\mathrm{T}}{\Delta^3} = \sqrt{\frac{e_1 - e_2}{G}}\ 8q\ \frac{1 - 2^2q^3 + 3^2q^8 - 4^2q^{15} + \text{etc.}}{(1 - 2q + 2q^4 - 2q^9 + \text{etc.})^3}$$

where q is to be derived as follows:—Let

$$\frac{e_1 - e_2}{e_1 - e_3} = \cos^2\theta \text{ and } \tfrac{1}{2}l = \left[\frac{\sin\frac{\theta}{2}}{1 + \sqrt{\cos\theta}}\right]^2$$

then

$$q = \tfrac{1}{2}l + 2(\tfrac{1}{2}l)^5 + 15(\tfrac{1}{2}l)^9 + 150(\tfrac{1}{2}l)^{13} + \text{etc.}$$

or in practice

$$\log q = \log \tfrac{1}{2}l + [9{\cdot}9388](\tfrac{1}{2}l)^4$$

(see *M. N.*, R.A.S., vol. lxii. p. 494).

All these formulæ are extraordinarily convergent. They can also be derived from the formulæ given on p. 333 of Hill's paper (*A.J.*, No. 511) in virtue of the relations

$$k = 4\sqrt{q}\left[\frac{1 + q^2 + q^6 + q^{12} + \text{etc.}}{1 + 2q + 2q^4 + 2q^9 + \text{etc.}}\right]^2 \qquad k_1 = \left[\frac{1 - 2q + 2q^4 - 2q^9 + \text{etc.}}{1 + 2q + 2q^4 + 2q^9 + \text{etc.}}\right]^2$$

$$\frac{2}{\pi}\int_0^{\frac{\pi}{2}} \frac{d\mathrm{T}}{\sqrt{1 - k^2\sin^2\mathrm{T}}} = (1 + 2q + 2q^4 + 2q^9 + \text{etc.})^2$$

$$\frac{2}{\pi}\int_0^{\frac{\pi}{2}} \sqrt{1 - k^2\sin^2\mathrm{T}}\, d\mathrm{T} = \left[\frac{1 + 3^2q^2 + 5^2q^6 + 7^2q^{12} + \text{etc.}}{1 - 3q^2 + 5q^6 - 7q^{12} + \text{etc.}}\right] \times \sqrt{k_1}$$

the last formula being one of some half dozen representations of Legendre's E by means of Jacobi's q function.

4.

Introducing the equations (*b*) into the equation (*a*) and putting

$$M = (e_2 - e_3)\Gamma_1 + (e_1 - e_3)\Gamma_2 + (e_2 - e_1)\Gamma_3$$
$$N = e_1(e_2 - e_3)\Gamma_1 + e_2(e_1 - e_3)\Gamma_2 + e_3(e_2 - e_1)\Gamma_3$$

enables us to write

$$\frac{1}{a^2}\frac{a}{r}R_0 = \frac{2}{\pi}\frac{1}{\sqrt{G}}\left[M\eta + N\omega\right] \quad . \quad . \quad . \quad (c)$$

with similar simple expressions for the two other components. If M and N are written in determinant form they are

$$M = \begin{vmatrix} \Gamma_1 & -\Gamma_2 & -\Gamma_3 \\ e_1 & e_2 & e_3 \\ 1 & 1 & 1 \end{vmatrix}. \qquad N = \begin{vmatrix} e_1\Gamma_1 & -e_2\Gamma_2 & -e_3\Gamma_3 \\ e_1 & e_2 & e_3 \\ 1 & 1 & 1 \end{vmatrix}$$

5.

We now proceed to the elimination of the roots of the cubic (e_1, e_2 and e_3) by means of the invariants g_2 and g_3. The following formulæ are collected here for easy reference:—

$$G_1 + G_2 - G_3 = k_1. \quad G_1G_2 - G_1G_3 - G_2G_3 = k_2. \quad -G_1G_2G_3 = k_3$$
$$\tfrac{3}{4}g_2 = \lambda = k_1^2 - 3k_2 = -3(e_1e_2 + e_1e_3 + e_2e_3)$$
$$\tfrac{1}{4}g_2 = \tfrac{1}{27}(2k_1^3 - 9k_1k_2 + 27k_3) = e_1e_2e_3$$
$$16G = g_2^3 - 27g_3^2 = 16(e_1 - e_2)(e_1 - e_3)(e_2 - e_3)$$

or for the last

$$\sqrt{G} = \tfrac{1}{4}g_3 \times \begin{vmatrix} 1, & 1, & 1 \\ \frac{1}{e_1} & \frac{1}{e_2} & \frac{1}{e_3} \\ e_1 & e_2 & e_3 \end{vmatrix} = \tfrac{1}{4}g_3 \times \begin{vmatrix} 1, & 1, & 1 \\ \frac{1}{e_1} & \frac{1}{e_2} & \frac{1}{e_3} \\ \frac{e_2e_3}{e_1} & \frac{e_1e_3}{e_2} & \frac{e_1e_2}{e_3} \end{vmatrix}$$

We have now four determinants; let us multiply the first by the third, and then the second by the fourth. The first result is written at length,

$$M\sqrt{G} = \begin{vmatrix} \Gamma_1 - \Gamma_2 - \Gamma_3, & e_2e_3\Gamma_1 - e_1e_3\Gamma_2 - e_1e_2\Gamma_3, & e_1\Gamma_1 - e_2\Gamma_2 - e_3\Gamma_3 \\ 0 & 3e_1e_2e_3 & -2(e_1e_2 + e_1e_3 + e_2e_3) \\ 3 & e_1e_2 + e_1e_3 + e_2e_3 & 0 \end{vmatrix}$$

To abbreviate put

$$\begin{aligned} \Gamma_1 - \quad \Gamma_2 - \quad \Gamma_3 &= \psi \\ e_2e_3\Gamma_1 - e_1e_3\Gamma_2 - e_1e_2\Gamma_3 &= \chi \\ e_1\Gamma_1 - \quad e_2\Gamma_2 - \quad e_3\Gamma_3 &= \phi \end{aligned} \quad . \quad . \quad . \quad (d)$$

and replace therein the constituents in e_1, e_2, and e_3 by their appropriate equivalents in g_2 and g_3; then expanding we obtain finally

$$M\sqrt{G} = \psi\tfrac{1}{8}g_2^2 + \tfrac{3}{2}g_2\chi - \tfrac{9}{4}g_3\phi$$

A similar process gives

$$N\sqrt{G} = -\psi\tfrac{3}{16}g_2g_3 - \tfrac{9}{4}g_3\chi + \tfrac{1}{8}g_2^2\phi$$

so that (*c*) becomes

$$\frac{1}{a^2}\frac{a}{r}R_0=\frac{2}{\pi}\frac{1}{\sqrt{G}}\left[\tfrac{1}{2}(g_2\eta-\tfrac{3}{2}g_3\omega)(\tfrac{1}{4}g_2\psi+3\chi)+\tfrac{1}{4}(\tfrac{1}{2}g_2^2\omega-9g_3\eta)\phi\right]$$

with similar equations for S_0 and W_0.

6.

Bruns first pointed out that the periods of the elliptic functions could be found without a knowledge of the three roots (H. Bruns, *Perioden der Ellip. Integrale*, Dorpat, 1875).

This remarkable investigation does not deserve the neglect it has received at the hands of English mathematicians. Bruns does not adapt his formulæ for numerical use, and I have found it better to adopt another absolute invariant. If we write

$$\cos\iota=3\sqrt{3}\frac{g_3}{g_2^{\frac{3}{2}}} \qquad (0^\circ<\iota<180^\circ)$$

we have

$$\frac{2}{\pi}\frac{1}{\sqrt{G}}\frac{1}{2}\left(g_2\eta-\frac{3}{2}g_3\omega\right)=\frac{5}{8}\frac{1}{\lambda^{\frac{1}{4}}}\frac{1}{\cos^2\frac{\iota}{2}}\,F\left(\frac{1}{6},\frac{5}{6},\,2.\,\sin^2\frac{\iota}{2}\right)$$

$$\frac{2}{\pi}\frac{1}{\sqrt{G}}\frac{1}{4}\left(\frac{1}{2}g_2^2\omega-9g_3\eta\right)=\frac{7}{8}\frac{1}{\lambda^{\frac{1}{4}}}\frac{\lambda^{\frac{1}{4}}}{\cos^2\frac{\iota}{2}}\,F\left(-\frac{1}{6},\frac{7}{6},\,2.\,\sin^2\frac{\iota}{2}\right)$$

Hence

$$\frac{1}{a^2}\frac{a}{r}R_0=\frac{1}{8}\frac{1}{\cos^2\frac{\iota}{2}}\frac{1}{\lambda^{\frac{1}{4}}}\left[\left(\frac{1}{4}g_2\psi+3\chi\right)5F\left(\frac{1}{6}\right)+\lambda^{\frac{1}{4}}\phi 7F\left(-\frac{1}{6}\right)\right]$$

with similar equations for S_0 and W_0. The abbreviations $F\left(\frac{1}{6}\right)$ and $F\left(-\frac{1}{6}\right)$ require no explanation.

7.

It remains to find the values of ψ, χ, and ϕ. Replacing the *e*'s by the G's in (*d*) we have

$$\psi=\Gamma_1-\Gamma_2-\Gamma_3.$$

$$\phi=\left(G_1-\frac{k_1}{3}\right)\Gamma_1-\left(G_2-\frac{k_1}{3}\right)\Gamma_2+\left(G_3+\frac{k_1}{3}\right)\Gamma_3=G_1\Gamma_1-G_2\Gamma_2+G_3\Gamma_3-\frac{k_1}{3}\psi$$

$$\chi=-G_2G_3\Gamma_1+G_1G_3\Gamma_2-G_1G_2\Gamma_3+\frac{k_1}{3}\phi-\left(\frac{k_1}{3}\right)^2\psi$$

Hill and Arndt give the formulæ for these quantities in G which are as follows:—

	$\frac{1}{a^2}\frac{a}{r}R_0$	$\frac{1}{a^2}\frac{a}{r}S_0$	$\frac{1}{a\sin J}\frac{a}{r}W_0$
$\Gamma_1-\Gamma_2-\Gamma_3 = \psi$	$-r_0$	0	0
$G_1\Gamma_1-G_2\Gamma_2+G_3\Gamma_3 = \phi+\frac{k_1}{3}\psi$	$-r_0(A_0-e_1B_0\cos\epsilon)+A_cD+A_sB_0\sin\epsilon$	$0+B_cD+B_sB_0\sin\epsilon$	$0+C_cD+C_sB_0\sin\epsilon$
$-G_2G_3\Gamma_1+G_1G_3\Gamma_2-G_1G_2\Gamma_3 = \chi-\frac{k_1}{3}\phi+\frac{k_1^2}{9}\psi$	$-e_1r_0B_0\sin\epsilon(A_sA_c-A_cA_s)=0$	$-e_1r_0B_0\sin\epsilon(B_sA_c-B_cA_s) = -e_1r_0B_0\sin\epsilon\cos\phi_1\cos J$	$-e_1r_0B_0\sin\epsilon(C_sA_c-C_cA_s) = -e_1r_0B_0\sin\epsilon\cos\phi_1\cos(\nu+\Pi)$

[where $D=(1+e_1^2)B_0\cos\epsilon-e_1(A_0+C_0)$].

These formulæ simplify in use. Working formulæ and two numerical applications follow.

Working Formulæ.

9.

Let

a = semi-axis major corrected for the constant part of its perturbations
n = mean motion
$e = \sin\phi$ = eccentricity
π = long. of perihelion from equinox
i = inclination of orbit to ecliptic
Ω = long. of ascending node
χ = long. of perihelion from a fixed point in orbit
$\omega = \pi - \Omega$
r = radius-vector
E = excentric anomaly
ν = true anomaly
m = mass (Sun's mass unity)

of the disturbed body; the addition of a suffix $(_1)$ indicates similar quantities appertaining to the disturbing body.

10.

$$\begin{aligned}
-\sin\tfrac{1}{2}\mathrm{J}\sin\tfrac{1}{2}(\Phi+\Psi) &= \sin\tfrac{1}{2}(\Omega_1-\Omega)\sin\tfrac{1}{2}(i_1+i)\\
-\sin\tfrac{1}{2}\mathrm{J}\cos\tfrac{1}{2}(\Phi+\Psi) &= \cos\tfrac{1}{2}(\Omega_1-\Omega)\sin\tfrac{1}{2}(i_1-i)\\
\cos\tfrac{1}{2}\mathrm{J}\sin\tfrac{1}{2}(\Phi-\Psi) &= \sin\tfrac{1}{2}(\Omega_1-\Omega)\cos\tfrac{1}{2}(i_1+i)\\
\cos\tfrac{1}{2}\mathrm{J}\cos\tfrac{1}{2}(\Phi-\Psi) &= \cos\tfrac{1}{2}(\Omega_1-\Omega)\cos\tfrac{1}{2}(i_1-i)
\end{aligned}$$

check formulæ

$$\begin{aligned}
\cos p\sin q &= \sin i_1\cos(\Omega_1-\Omega)\\
\cos p\cos q &= \cos i_1\\
\cos p\sin r &= -\cos i_1\sin(\Omega_1-\Omega)\\
\cos p\cos r &= \cos(\Omega_1-\Omega)\\
\sin p &= -\sin i_1\sin(\Omega_1-\Omega)
\end{aligned}$$

then

$$\begin{aligned}
\sin\mathrm{J}\sin\Phi &= \sin p\\
\sin\mathrm{J}\cos\Phi &= \cos p\sin(i-q)\\
\sin\mathrm{J}\sin(\Psi-r) &= \sin p\cos(i-q)\\
\sin\mathrm{J}\cos(\Psi-r) &= \sin(i-q)\\
\cos\mathrm{J} &= \cos p\cos(i-q)
\end{aligned}$$

$$\Pi = \omega - \Phi\,. \qquad \Pi_1 = \omega_1 - \Psi$$

$$\begin{aligned}
k\cos(\mathrm{K}-\Pi) &= \cos\Pi_1\,. & k_1\cos(\mathrm{K}_1-\Pi) &= \cos\Pi_1\cos\mathrm{J}\\
k\sin(\mathrm{K}-\Pi) &= -\sin\Pi_1\cos\mathrm{J}\,. & k_1\sin(\mathrm{K}_1-\Pi) &= -\sin\Pi_1
\end{aligned}$$

check: $k^2 + k_1^2 = 1 + \cos^2\mathrm{J}$

When the Earth is the disturbing planet, these equations simplify to

$$\begin{aligned}
\mathrm{J} &= i\\
\Phi &= 0\\
\Pi &= \pi - \Omega\\
\Pi_1 &= \pi_1 - \Omega
\end{aligned}$$

and when the Earth is the disturbed planet, to

$$J = i_1$$
$$\Psi = 180^\circ$$
$$\Pi = 180^\circ + \pi - \Omega_1$$
$$\Pi_1 = 180^\circ + \pi_1 - \Omega_1$$

II.

The various functions of E which follow should now be computed for $4j$ equal parts of the circumference. The larger the number of parts is, the more exact will be the results of the mechanical integration, but j should not be taken larger than necessary. A measure of the accuracy attained is given by adding the odd values and the even values of each of the quantities: if these differ, more values of E are required. Reference to Hill's original example or to those of later computers will assist the judgment.

Take $a = a/a_1$, then omitting the suffix $_0$ used in the demonstration, when no confusion will be created thereby, we have

$$r_0 \cos \nu = a(\cos E - e). \qquad r_0 \sin \nu = a \cos \phi \sin E$$
$$r_0 = a(1 - e \cos E). \text{ (check)}$$
$$k \cos (\nu + K) = A_c \qquad k_1 \cos \phi_1 \sin (\nu + K_1) = A_s$$
$$-k \sin (\nu + K) = B_c \qquad k_1 \cos \phi_1 \cos (\nu + K_1) = B_s$$
$$A = 1 + r_0^2 + 2e_1 r_0 A_c.$$
$$B \cos \epsilon = r_0 A_c + e_1. \qquad B \sin \epsilon = r_0 A_s$$
$$k_1 = A - e_1^2. \qquad k_2 = B_2 - A e_1^2. \qquad -k_3 = e_1^2 B^2 \sin^2 \epsilon$$
$$\tfrac{3}{4} g_2 = \lambda = k_1^2 - 3k_2 \qquad g_3 = \tfrac{4}{27}(2k_1^3 - 9k_1 k_2 + 27k_3)$$
$$\cos \iota = \frac{\sqrt{27} g_3}{g_2^{\frac{3}{2}}}. \qquad (0^\circ < \iota < 180^\circ)$$
$$D = (1 + e_1^2) B \cos \epsilon - e_1 (A + e_1^2)$$
$$\phi_R = -r_0 \left(A - e_1 B \cos \epsilon - \frac{k_1}{3} \right) + A_c D + A_s B \sin \epsilon$$
$$\phi_S = + B_c D + B_s B \sin \epsilon$$
$$\phi_W = + \sin \Pi_1 D + \cos \phi_1 \cos \Pi_1 B \sin \epsilon$$
$$\chi_R - \frac{k_1}{3} \phi_R - \frac{k_1^2}{9} r_0 = 0$$
$$\chi_S - \frac{k_1}{3} \phi_S = -e_1 r_0 B \sin \epsilon \cos \phi_1 \cos J$$
$$\chi_W - \frac{k_1}{3} \phi_W = -e_1 r_0 B \sin \epsilon \cos \phi_1 \cos (\nu + \Pi)$$

With the aid of the tables or formulæ compute

$$\frac{5}{8}\frac{1}{\cos^2\frac{\iota}{2}}\,\mathrm{F}\left(1/6\,,\,5/6\,,\,2\,,\,\sin^2\frac{\iota}{2}\right)=\mathrm{F_A}$$

and

$$\frac{7}{8}\frac{1}{\cos^2\frac{\iota}{2}}\mathrm{F}\left(-1/6\,,\,7/6\,,\,2\,,\,\sin^2\frac{\iota}{2}\right)=\mathrm{F_B}$$

Then

$$\frac{1}{\alpha_2}\frac{a}{r}\mathrm{R}_0=\left(3\chi-\frac{1}{4}g_2r_0\right)\frac{\mathrm{F_B}}{\lambda^{\frac{7}{4}}}+\frac{\mathrm{F_A}}{\lambda^{\frac{5}{4}}}$$

$$\frac{1}{\alpha_2}\frac{a}{r}\mathrm{S}_0=(\quad 3\chi\quad)\ ,,\ ,,\ ,,$$

$$\frac{1}{\alpha}\frac{1}{\sin\mathrm{J}}\frac{a}{r}\mathrm{W}_0=[(\quad 3\chi\quad)\ ,,\ ,,\ ,,]r_0$$

12.

With the $4j$ values of these three functions, which may be designated

$$\mathrm{R}^{(0)},\ \mathrm{R}^{(1)},\ \mathrm{R}^{(2)},\ \ldots\ \mathrm{R}^{(4j-1)}$$
$$\mathrm{S}^{(0)},\ \mathrm{S}^{(1)},\ \ldots\ \mathrm{S}^{(4j-1)}$$
$$\mathrm{W}^{(0)},\ \mathrm{W}^{(1)},\ \ldots\ \mathrm{W}^{(4j-1)}$$

compute

$$\mathrm{A}_0^{(c)}=\frac{1}{4j}\left[\mathrm{R}^{(0)}+\mathrm{R}^{(1)}+\mathrm{R}^{(2)}\ \ldots\ \mathrm{R}^{(4j-1)}\right]$$

$$\frac{1}{2}\mathrm{A}_1^{(c)}=\frac{1}{4j}\left[\mathrm{R}^{(0)}+\mathrm{R}^{(1)}\cos\frac{1}{j}\frac{\pi}{2}+\mathrm{R}^{(2)}\cos\frac{2}{j}\frac{\pi}{2}\ \ldots\ +\mathrm{R}^{(4j-1)}\cos\frac{4j-1}{j}\frac{\pi}{2}\right]$$

$$\frac{1}{2}\mathrm{A}^{(s)}=\frac{1}{4j}\left[\mathrm{R}^{(1)}\sin\frac{1}{j}\frac{\pi}{2}+\mathrm{R}^{(2)}\sin\frac{2}{j}\frac{\pi}{2}\ \ldots\ +\mathrm{R}^{(4j-1)}\sin\frac{4j-1}{j}\frac{\pi}{2}\right]$$

$$\frac{1}{2}\mathrm{A}_2^{(c)}=\frac{1}{4j}\left[\mathrm{R}^{(0)}+\mathrm{R}^{(1)}\cos\frac{2}{j}\frac{\pi}{2}+\mathrm{R}^{(2)}\cos\frac{4}{j}\frac{\pi}{2}\ \ldots\ +\mathrm{R}^{(4j-1)}\cos\frac{2(4j-1)}{j}\frac{\pi}{2}\right]$$

$$\frac{1}{2}\mathrm{A}_2^{(s)}=\frac{1}{4j}\left[\mathrm{R}^{(1)}\sin\frac{2}{j}\frac{\pi}{2}+\mathrm{R}^{(2)}\sin\frac{4}{j}\frac{\pi}{2}\ \ldots\ +\mathrm{R}^{(4j-1)}\sin\frac{2(4j-1)}{j}\frac{\pi}{2}\right]$$

with similar equations for $\mathrm{B}_0^{(c)}$ $\mathrm{B}_1^{(c)}$ . . . . and $\mathrm{C}_0^{(c)}$ $\mathrm{C}_1^{(c}$ and $\mathrm{C}_1^{(s)}$.

(Note :— $\mathrm{A}_2^{(s)}$ is not required.)

For any special case of j these equations become greatly simplified: thus for $j=2$ we have

$$\mathrm{A}_0^{(c)}=\tfrac{1}{8}\left[\mathrm{R}^{(0)}+\mathrm{R}^{(1)}+\mathrm{R}^{(2)}\ \ldots\ +\mathrm{R}^{(7)}\right]$$
$$\tfrac{1}{2}\mathrm{A}_1^{(c)}=\tfrac{1}{8}\left[\mathrm{R}^{(0)}-\mathrm{R}^{(4)}+(\mathrm{R}^{(1)}-\mathrm{R}^{(3)}-\mathrm{R}^{(5)}+\mathrm{R}^{(7)})\cos 45^\circ\right]$$
$$\tfrac{1}{2}\mathrm{A}_1^{(s)}=\tfrac{1}{8}\left[\mathrm{R}^{(2)}-\mathrm{R}^{(6)}+(\mathrm{R}^{(1)}+\mathrm{R}^{(3)}-\mathrm{R}^{(5)}-\mathrm{R}^{(7)})\sin 45^\circ\right]$$
$$\tfrac{1}{2}\mathrm{A}_2^{(c)}=\tfrac{1}{8}\left[\mathrm{R}^{(0)}-\mathrm{R}^{(2)}+\mathrm{R}^{(4)}-\mathrm{R}^{(6)}\right]$$
$$\tfrac{1}{2}\mathrm{A}_2^{(s)}=\tfrac{1}{8}\left[\mathrm{R}^{(1)}-\mathrm{R}^{(3)}+\mathrm{R}^{(4)}-\mathrm{R}^{(6)}\right]$$

and so on. For other values of j the equations are easily formed, or may be found in Hansen's *Auseinandersetzung* or elsewhere.

The equation

$$\sin\phi\tfrac{1}{2}A_1^{(s)} + \cos\phi B_0^{(c)} = 0$$

affords a useful check.

13.

The final equations are

$$e\left[\frac{d\phi}{dt}\right]_{00} = \frac{m_1 n}{1+m}\,a^2\left[-\left(1+\frac{e^2}{2}\right)B_0^{(c)} - 2e\,\frac{1}{2}\,B_1^{(c)} + \frac{e^2}{2}\,\frac{1}{2}\,B_2^{(c)}\right]$$

$$e\left[\frac{d\chi}{dt}\right]_{00} = \frac{m_1 n}{1+m}\,a_2\left[eA_0^{(c)}\cos\phi - \frac{1}{2}A_1^{(c)}\cos\phi + (2-e^2)\,\frac{1}{2}\,B_1^{(s)} - \frac{e}{2}\,\frac{1}{2}\,B_2^{(s)}\right]$$

$$\left[\frac{di}{dt}\right]_{00} = \frac{m_1 n}{1+m}\,a\sin J\cos\omega\left[\left(\frac{1}{2}C_1^{(c)} - eC_0^{(c)}\right)\sec\phi - \frac{1}{2}C_1^{(s)}\tan\omega\right]$$

$$\sin i\left[\frac{d\Omega}{dt}\right]_{00} = \frac{m_1 n}{1+m}\,a\sin J\cos\omega\left[\left(\frac{1}{2}C_1^{(c)} - eC_0^{(c)}\right)\sec\phi\tan\omega + \frac{1}{2}C_1^{(s)}\right]$$

$$\left[-2\frac{r}{a}R_0\right]_{00} = a^2\left[-(2+e^2)A_0^{(c)} + 4e\,\frac{1}{2}\,A_1^{(c)} - e^2\,\frac{1}{2}\,A_2^{(c)}\right]$$

Calculation of $F\left(\frac{1}{6}, \frac{5}{6}, 2, \sin^2\frac{\iota}{2}\right)$ and $F\left(-\frac{1}{6}, \frac{7}{6}, 2, \sin^2\frac{\iota}{2}\right)$

14.

When ι is less than 90° the calculation of these series is not long, but if an accuracy of two units in the 7th place of the decimals of the logarithms is sufficient, the use of series can be avoided. This accuracy is indeed greater than either our knowledge of the planetary masses or the precision of observations requires. Besides these, the change in the secular perturbations due to higher powers of the masses than the first is of a more important character.

The approximate formulæ are

$$\log F\left(\frac{1}{6}, \frac{5}{6}, 2.\ \sin^2\frac{\iota}{2}\right) = \log\left\{1 + \frac{\frac{5}{72}\sin^2\frac{\iota}{2}}{\left(\sqrt{1-\frac{2}{3}\sin^2\frac{\iota}{2}}\right)^{1+\frac{5}{72}}}\right\} + [6{\cdot}0049]\left(\frac{\iota^\circ}{100}\right)^{\frac{100}{12}}$$

$$\log F\left(-\frac{1}{6}, \frac{7}{6}, 2.\ \sin^2\frac{\iota}{2}\right) = \log\left\{1 - \frac{\frac{7}{72}\sin^2\frac{\iota}{2}}{\left(\sqrt{1-\frac{2}{3}\sin^2\frac{\iota}{2}}\right)^{1-\frac{7}{72}}}\right\} - [6{\cdot}1011]\left(\frac{\iota^\circ}{100}\right)^{\frac{100}{12}}$$

The two series are inert :—

ι	$F\left(\frac{1}{6}, \frac{5}{6}, 2, \sin^2\frac{\iota}{2}\right)$	$F\left(-\frac{1}{6}, \frac{7}{6}, 2, \sin^2\frac{\iota}{2}\right)$
0°	1·0000000	1·0000000
90°	0·9415120	1·0432298
180°	$0{\cdot}8185111 = \frac{2}{\pi}\frac{9}{7}$	$1{\cdot}1459156 = \frac{2}{\pi}\frac{9}{5}$

If ι exceeds 90° the best procedure seems to be as follows :—

Take $\iota_1 = 180° - \iota$,

then proceeding at once to the actual quantities wanted and writing

$$H = \frac{2}{\pi}\frac{108}{\sin^2\frac{\iota_1}{2}} \frac{1}{\left[7F\left(-\frac{1}{6}, \frac{7}{6}, 2, \sin^2\frac{\iota_1}{2}\right) + 5\cos\iota_1 F\left(\frac{1}{6}, \frac{5}{6}, 2, \sin^2\frac{\iota_1}{2}\right)\right]}$$

we have

$$\frac{5}{\cos^2\frac{\iota}{2}} F\left(\frac{1}{6}, \frac{5}{6}, 2, \sin^2\frac{\iota}{2}\right) = H - \frac{1}{\pi}\log_e\frac{1}{q}\frac{5}{\cos^2\frac{\iota_1}{2}} F\left(\frac{1}{6}, \frac{5}{6}, 2, \sin^2\frac{\iota_1}{2}\right)$$

$$\frac{7}{\cos^2\frac{\iota}{2}} F\left(-\frac{1}{6}, \frac{7}{6}, 2, \sin^2\frac{\iota}{2}\right) = H\cos\iota_1 + \frac{1}{\pi}\log_e\frac{1}{q}\frac{7}{\cos^2\frac{\iota_1}{2}} F\left(-\frac{1}{6}, \frac{7}{6}, 2, \sin^2\frac{\iota_1}{2}\right)$$

To compute q put

$$\frac{1}{\sqrt{3}}\tan\frac{\iota_1}{3} = \cos 2\gamma$$

and $$\sqrt{\tan\gamma} = \cos 2\lambda$$

then $$l = \tan^2\lambda$$

and $$q = \tfrac{1}{2}l + 2(\tfrac{1}{2}l)^5 + \text{etc.}$$

In practice it may be assumed that $\log q = \log \frac{1}{2}l$. This assumption is in defect by the following quantities :—

ι_1	Correction to $\log \frac{1}{2}l$ in units of the 7th place of decimals,
0° – 40°	+0
40°	1
50°	2
60°	4
70°	9
80°	17
90°	+30

As $\log_e \frac{1}{q}$ is the natural log of $\frac{1}{q}$, we have in ordinary logarithms—

$$\log\left(\frac{1}{\pi}\log_e\frac{1}{q}\right) = \log \text{ of } \log\frac{1}{q} + 9{\cdot}8650658$$

15.

Numerical Examples.

In the *Astron. Nachr.*, No. 4068, Dr Arthur B. Turner has computed the secular perturbations of Mars by Jupiter using Arndt's formulæ.

The preceding formulæ are applied to this case for the case of $E = 0°$. We have $J = 1°26'$, $\Pi = 149°{\cdot}8$, $\Pi_1 = 188°{\cdot}4$, K and $K_1 = 321°{\cdot}4$, k and $k_1 = 1$, $\log r_0 = 9{\cdot}4241$, $\nu = 0°$, $\log A_c = 9{\cdot}893$, $\log B_c = 9{\cdot}795$, $\log A_s = 9{\cdot}794n$, $\log B_s = 9{\cdot}892$, $A_0 = 1{\cdot}09$, $\log B_0 \sin\epsilon = 9{\cdot}219$, $k_1 = 1{\cdot}088$, $k_2 = 0{\cdot}090$, $k_3 = -0{\cdot}000$, $g_2 = 1{\cdot}218$, $g_3 = 0{\cdot}251$, $\iota = 14°20'$, $F(\frac{1}{6}) = 0{\cdot}998$, $F(-\frac{1}{6}) = 1{\cdot}001$, $D = 0{\cdot}204$, and

	R_0	S_0	W_0
ϕ	0·072	0·002	0·134
χ	0·061	0·001	0·050

With these figures Turner's results are reproduced.

The next example is taken from Herr W. Dziewulski's paper on the secular perturbations of Eros by Mars. In the present position of the orbits for $E = 120°$, the modular angle $\theta = 58°{\cdot}9$. In the notation of the present paper we have $J = 11°{\cdot}5$, $\Pi = 6°{\cdot}7$, $\Pi_1 = 219°{\cdot}6$, $K = 147°{\cdot}6$, $K_1 = 146°{\cdot}5$, $\log k = 9{\cdot}996$, $\log k_1 = 9{\cdot}995$, $\log r_0 = 0{\cdot}0268$, $\nu = 130°{\cdot}6$, $\log A_c = 9{\cdot}152$, $\log B_c = 9{\cdot}992$, $\log A_s = 9{\cdot}990n$, $\log B_s = 9{\cdot}084$, $A_0 = 2{\cdot}16$, $\log B_0 \sin\epsilon = 0{\cdot}016n$, $k_1 = 2{\cdot}151$, $k_2 = 1{\cdot}120$, $k_3 = -0{\cdot}009$, $g_2 = 1{\cdot}690$, $g_3 = -0{\cdot}300$, $\iota = 135°{\cdot}1$, $\iota_1 = 44°{\cdot}9$, $\gamma = 40°{\cdot}6$, $\lambda = 11°{\cdot}1$, $\log \frac{1}{2}l = 8{\cdot}288$, $\log \frac{1}{\pi}\log_e \frac{1}{q} = 0{\cdot}099$, $F_1(\frac{1}{6}) = 0{\cdot}985$, $F_1(-\frac{1}{6}) = 1{\cdot}011$, $F(\frac{1}{6}) = 0{\cdot}881$, $F(-\frac{1}{6}) = 1{\cdot}102$, $D = 0{\cdot}044$, and

	R_0	S_0	W_0
ϕ	−0·490	−0·083	0·763
χ	0·195	0·041	0·472

leading to Dziewulski's results.

[For Tables, see paper by Mr Robbins.]

Johannesburg.

Tables for the Application of Mr Innes's Method.

By Frank Robbins.

As an appendix to the preceding paper on "The Computation of Secular Perturbations," the author asked me to compute for each degree of the quadrant the logarithmic values (base 10) of the two functions of *iota* required for the convenient application of his method.

In the hypergeometric series F $(\alpha\ \beta\ \gamma\ x)$ in the first case

$$\alpha \text{ has the value } -\frac{1}{6} \quad \beta=\frac{7}{6} \quad \gamma=2 \quad x=\sin^2\frac{\iota}{2}$$

and in the second case

$$\alpha=\frac{1}{6} \quad \beta=\frac{5}{6} \quad \gamma=2 \quad x=\sin^2\frac{\iota}{2}$$

For convenience of designation the tables are headed *Minus* F and *Plus* F according to the sign of α.

Vega's (1794) ten-figure logarithms, corrected by collation with the copy in use at H.M. *Nautical Almanac* Office, were used, and the natural values of the individual terms were taken out to ten places of decimals. These were obtained in duplicate for each end of the quadrant, and the whole were examined by differencing to the sixth order. Lastly, the seven-figure logarithms of the functions were taken from the eight-figure table of the Service Géographique de l'Armée (Paris, 1891), reference being made to Vega where the eighth figure was approximately five.

The log *Minus* F has been increased by 10 as customary, to avoid the inconvenience of printing negative characteristics.

The whole has been examined by Mr J. Abner Sprigge, of H.M. *Nautical Almanac* Office, so as to make it possible to use the tables with confidence in their accuracy to the seventh place.

(Iota).	Log plus F.	Δ_1	Δ_2	Log minus F.	Δ_1	Δ_2
°						
1	0·0000023			9·9999968		
		+ 69			− 97	
2	0092		+46	9871		−63
		115			160	
3	0207		45	9711		65
		160			225	
4	0367		47	9486		65
		207			290	
5	0574		46	9196		64
		253			354	
6	0827		46	8842		64
		299			418	
7	1126		45	8424		64
		344			482	
8	0·0001470		+46	9·9997942		−65
		+390			−547	

(Iota).	Log plus F.	Δ_1	Δ_2	Log minus F.	Δ_1	Δ_2
9°	0·0001860		+47	9·9997395		−64
		+437			−611	
10	2297		45	6784		64
		482			675	
11	2779		46	6109		65
		528			740	
12	3307		46	5369		64
		574			804	
13	3881		46	4565		65
		620			869	
14	4501		46	3696		64
		666			933	
15	5167		45	2763		64
		711			997	
16	5878		47	1766		65
		758			1062	
17	6636		45	9·9990704		64
		803			1126	
18	7439		46	9·9989578		65
		849			1191	
19	8288		47	88387		64
		896			1255	
20	0·0009184		45	87132		65
		941			1320	
21	0·0010125		46	85812		64
		987			1384	
22	11112		45	84428		65
		1032			1449	
23	12144		47	82979		64
		1079			1513	
24	13223		45	81466		65
		1124			1578	
25	14347		46	79888		64
		1170			1642	
26	15517		45	78246		65
		1215			1707	
27	16732		46	76539		64
		1261			1771	
28	17993		46	74768		66
		1307			1837	
29	19300		46	72931		64
		1353			1901	
30	20653		46	71030		64
		1399			1965	
31	22052		45	69065		66
		1444			2031	
32	23496		46	67034		64
		1490			2095	
33	24986		45	64939		65
		1535			2160	
34	26521		46	62779		64
		1581			2224	
35	28102		46	60555		66
		1627			2290	
36	29729		45	58265		64
		1672			2354	
37	31401		46	55911		65
		1718			2419	
38	0·0033119		+45	9·9953492		−65
		+1763			−2484	

(Iota).	Log plus F.	Δ_1	Δ_2	Log minus F.	Δ_1	Δ_2
39	0·0034882			9·9951008		−65
		+1808	+45		−2549	
40	36690		46	48459		64
		1854			2613	
41	38544		46	45846		66
		1900			2679	
42	40444		45	43167		65
		1945			2744	
43	42389		45	40423		64
		1990			2808	
44	44379		45	37615		66
		2035			2874	
45	46414		46	34741		64
		2081			2938	
46	48495		44	31803		66
		2125			3004	
47	50620		46	28799		64
		2171			3068	
48	52791		46	25731		66
		2217			3134	
49	55008		44	22597		65
		2261			3199	
50	57269		45	19398		65
		2306			3264	
51	59575		45	16134		64
		2351			3328	
52	61926		45	12806		66
		2396			3394	
53	64322		45	09412		65
		2441			3459	
54	66763		45	05953		66
		2486			3525	
55	69249		45	9·9902428		64
		2531			3589	
56	71780		44	9·9898839		66
		2575			3655	
57	74355		45	95184		64
		2620			3719	
58	76975		45	91465		66
		2665			3785	
59	79640		44	87680		65
		2709			3850	
60	82349		44	83830		65
		2753			3915	
61	85102		45	79915		65
		2798			3980	
62	87900		44	75935		65
		2842			4045	
63	90742		44	71890		65
		2886			4110	
64	93628		44	67780		65
		2930			4175	
65	96558		44	63605		65
		2974			4240	
66	0·0099532		45	59365		65
		3019			4305	
67	0·0102551		43	55060		65
		3062			4370	
68	0·0105613		+43	9·9850690		−65
		+3105			−4435	

(Iota).	Log plus F.	Δ_1	Δ_2	Log minus F.	Δ_1	Δ_2
69	0·0108718		+45	9·9846255		−65
		+3150			−4500	
70	111868		42	41755		65
		3192			4565	
71	115060		44	37190		64
		3236			4629	
72	118296		43	32561		65
		3279			4694	
73	121575		44	27867		65
		3323			4759	
74	124898		43	23108		64
		3366			4823	
75	128264		42	18285		66
		3408			4889	
76	131672		43	13396		63
		3451			4952	
77	135123		43	08444		64
		3494			5016	
78	138617		42	9·99803428		65
		3536			5081	
79	142153		42	9·99798347		64
		3578			5145	
80	145731		43	93202		64
		3621			5209	
81	149352		41	87993		64
		3662			5273	
82	153014		42	82720		63
		3704			5336	
83	156718		42	77384		64
		3746			5400	
84	160464		41	71984		64
		3787			5464	
85	164251		42	66520		62
		3829			5526	
86	168080		40	60994		64
		3869			5590	
87	171949		40	55404		62
		3909			5652	
88	175858		42	49752		64
		3951			5716	
89	179809		+40	44036		−62
		+3991			−5778	
90	0·0183800			9·9738258		

Description of the 30-inch Photographic Reflector of the Helwân Observatory. By J. H. Reynolds.

There has recently been added to the equipment of the Khedivial Observatory at Helwân, near Cairo, a photographic equatorial reflector of 30-inch aperture: before giving a description of this instrument, a little explanation as to its origin would not be out of place. About five years ago one of the 30-inch Mirrors of Standard Astrographic focal length, which were ground and figured by the late Dr Common, came into my hands. I originally purposed to employ the mirror for nebular photography in this country, and commenced designing a suitable mounting. A visit to Egypt in

1902, however, impressed on me the great advantages of a good climate for such work, and in January 1904 I went out again with the object of seeing if any arrangements could be made for setting the instrument up in Egypt and for working it.

I found an equatorial house and dome of 35 feet diameter had been erected on the hills above Helwân, about 12 miles south of Cairo: this dome was originally intended for the old 8-inch equatorial refractor, but it was much larger than was necessary for this instrument, and has proved of suitable size for the 30-inch Reflector.

The Director-General of the Survey Department, Captain H. G. Lyons, R.E., D.Sc., F.R.S., agreed, on behalf of the Egyptian Government, to accept the instrument for the Khedivial Observatory under certain conditions as to its employment, etc., and I have to express my thanks to him for the great assistance he has rendered in bringing the project to a successful issue.

The design of the mounting was commenced in June 1904, and the heavy parts were shipped and erected in the autumn of 1905. It was not until the beginning of this year, however, that photographic work was commenced, and the capabilities of the instrument tested.

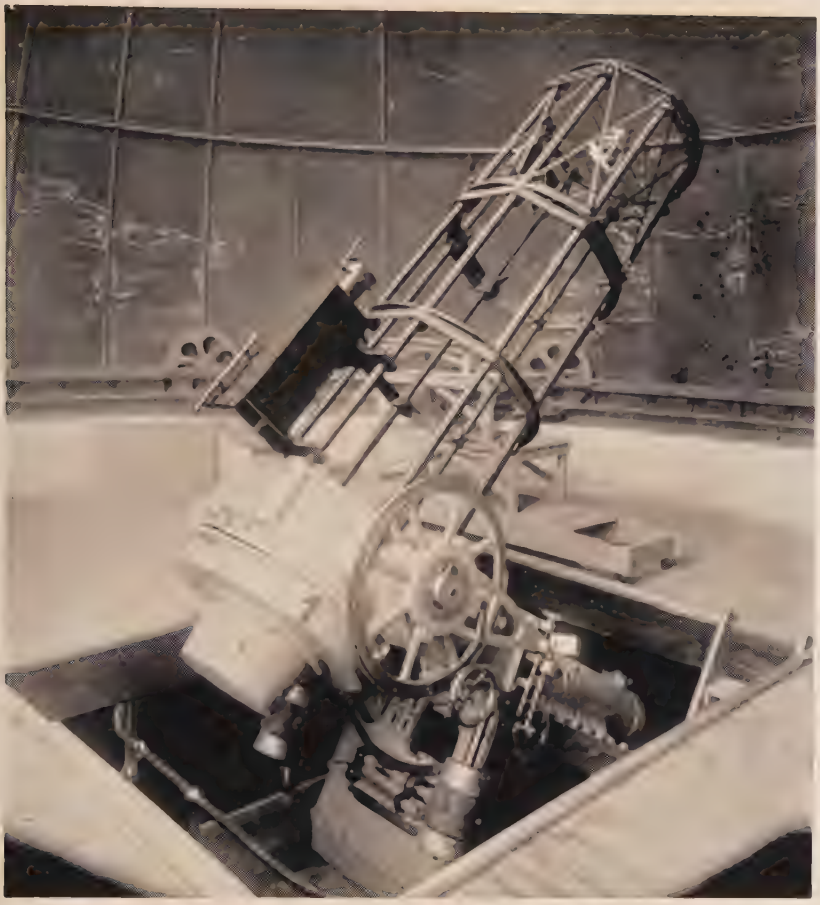
The angle block carrying the polar axis is supported on a cast-iron base 7 feet high, which is in two sections, and rests on a concrete pier having a foundation of limestone. To this pier the base is securely attached by means of holding-down bolts.

Round the top of the base runs a steel gallery giving access to the driving and setting mechanism. For the convenience of the operator all the motions and clamps are on the west side of the mounting, so that the setting of the telescope can be done completely from one position.

The angle block is provided with adjusting screws in azimuth and latitude, and contains the driving clock.

The polar axis, which is 6 feet long, is of mild steel and tapers from 6 inches diameter at the top to 5 inches at the lower end. It is held in position by two solid bronze bearings mounted in a cast-iron sleeve: two rings of 1-inch ball-bearings at the upper end of the sleeve and four rings at the lower end are so adjusted as to take the transverse and longitudinal thrusts on the polar axis. This manner of reducing the friction is very efficient, and the motion of the axis, which carries almost two tons, leaves nothing to be desired.

Attached by means of a flange to the upper end of the polar axis is the hollow cast-iron fork which carries the telescope, which has been made of considerable strength to prevent flexure. Some difficulty was experienced in designing this part of the mounting, as it was found that if the declination axis was placed in the same plane as the polar axis, the telescope could not be utilised far enough to the south, owing to the low latitude. It was ultimately decided to raise the declination axis vertically so as to allow of the telescope reaching to 40° south declination, the overbalancing east and west of the meridian which would result being counterbalanced



The 30-Inch Reflector of the Helwàn Observatory.

by weights at the back of the fork. The further south one gets, the more pronounced this difficulty becomes, and I do not think it would be possible to employ the open-fork type of mounting at any lower latitude in consequence.

The telescope itself consists of an octagonal cast-iron base to which the mirror cell is bolted, and a light framework tube of seamless steel tubes and aluminium rings, similar to that of the 24-inch Reflector of the Yerkes Observatory: the cell and mirror supports were constructed by the late Dr Common, and have been added to the telescope without alteration.

The flat mounting is of aluminium, to lighten the upper end of the telescope as far as possible, and this, with the photographic and optical apparatus, is contained in a separate section.

The following is done by means of a positive eyepiece with electrically-illuminated double cross wires, placed as near the photographic dark slide as possible: the double stage, with screw motions in two directions at right angles to each other, is identical in arrangement with that originated and described by Dr Common. The driving clock is provided with maintaining gear and has frictional governors of the cross-armed American type: the winding gear and starting screw are both accessible from the west side of the gallery. The driving arc runs for four hours continuously and can quickly be brought back to the starting position: differential gear for lunar rate is also provided.

In actual working this method of mounting a reflector is found to be convenient, and the clock drives at a uniform rate. There are of course one or two small defects still to be remedied, as we have only been able to work the instrument for a week or so, but we have sufficiently tested it to know that when it is in thorough working order there is no reason why satisfactory results should not be achieved.

The erection of the instrument at the Helwân Observatory was done under the superintendence of Mr B. F. E. Keeling of the Survey Department, and has been accomplished, in spite of many difficulties, in a most admirable manner. Mr Keeling is also responsible for adapting the equatorial house to the requirements of a reflector, and a false floor has been erected round the instrument near the level of the sill plate of the dome. An adjustable observing platform has also been constructed, which has proved to be very suitable for reaching the eyepiece in all positions.

The work primarily to be undertaken with the instrument is nebular photography, especially of the region lying between the equator and 40° south declination. This region includes many interesting objects which, I think, have not yet been photographed with the light grasp of such an instrument. It is intended to add a finder of good aperture, so that it may be employed on cometary photography: one piece of work which would be very suited to its capabilities is a search for Halley's Comet, as was recently suggested by Mr Crommelin.

The climate of Egypt is very suitable for astronomical photography during the greater part of the year: at times the atmosphere is so clear that stars are seen to set suddenly behind the desert horizon, and from results already obtained we know the air is extraordinarily transparent to actinic rays. Optically the mirrors are of very fair figure, especially considering that the large parabolic mirror lay in a packing-case for several years; and I am glad to have been able to mount one of Dr Common's mirrors in such a favourable situation. As far as I am aware, the Crossley Reflector of the Lick Observatory is the only reflector in similar latitudes, and almost all of this instrument was Dr Common's work.

In conclusion, I should like to thank Professor Turner for the kind encouragement and advice he has given in what was to me a rather unaccustomed task, and also Mr E. I. Jenkins for his assistance with the design.

REFERENCES TO FIGURE. (Plate 2.)

A. Concrete pier built up on limestone foundation with well for clock-weight 5 feet deep.

B^1 B^2. Cast-iron base 7 feet high attached by means of holding-down bolts to concrete pier. The openings are filled in with plate glass, and a steel door on the south side of B^1 gives access to the interior of the base. B^3 is bolted down to B^2, and has adjusting screws for latitude and azimuth.

CC. Steel gallery supported on cast iron brackets attached to B^2.

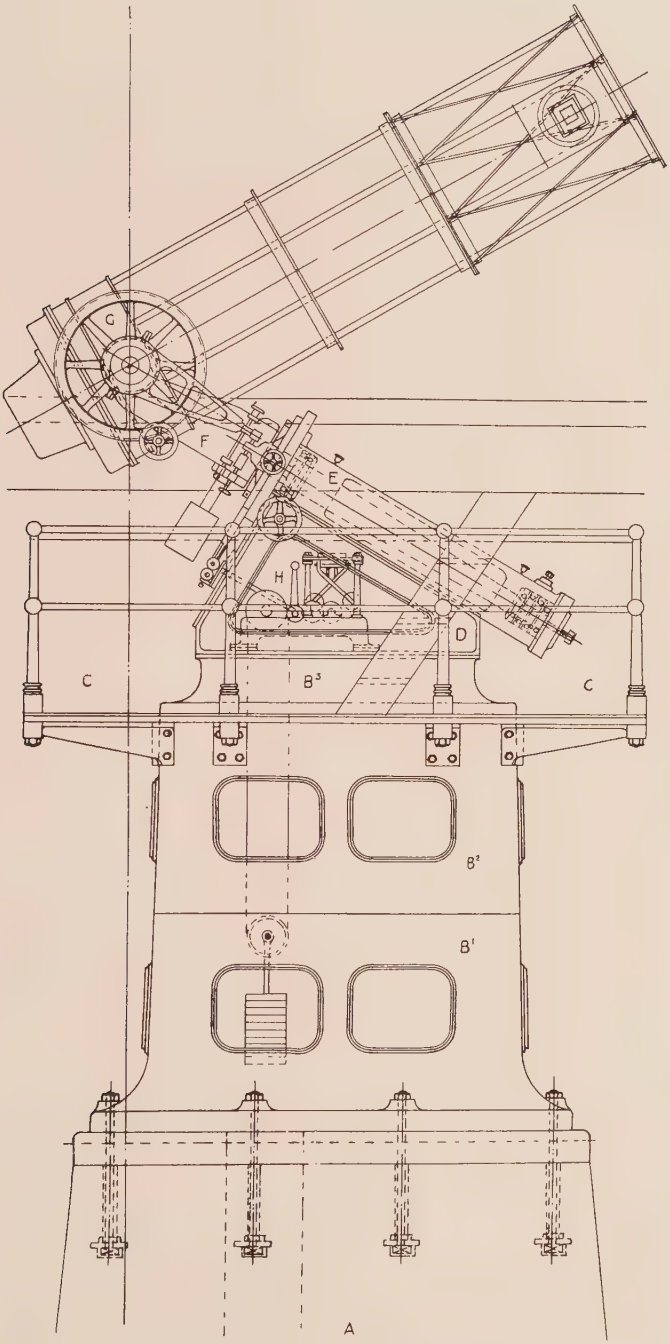
D. Angle block made in four sections bolted together, having large openings on all sides except the south, for easy access to the driving clock. The clock-winding gear is attached to the west side of D, which also carries the hand wheel and gear for the R.A. quick motion.

E. Heavy cast-iron sleeve bolted to D with solid bronze bearings and six ball-bearing rings, by means of which the weight of the polar axis, etc., is taken off the solid bearings. To the top of E is attached a flang plate for the R.A. adjustable vernier. The R.A. circle ring is bolted to a cast-iron plate keyed on to the polar axis: a rack is cut on the interior of this ring for the R.A. quick-motion pinion. The driving sector runs loose on the polar axis, and is cut with teeth 10 to the inch: on the top of the sector is a V-shaped wheel, to which the R.A. slow-motion arm is clamped.

F. Hollow cast-iron fork in which the telescope is suspended on trunnions: a plate on the west side holds the bearings for the slow-motion screws which pass through nuts let into the slow-motion arms: a double Hook's joint carries the motion of the R.A. clamp and treminates in a wheel.

G. Octagonal cast iron base of telescope tube: the trunnions are of steel bolted on to G. The west trunnion carries the declination circle: a pinion and hand wheel fixed on to the back of F give the declination quick motion. The mirror cell is bolted on to G, and is provided with adjusting screws: at the back of the cell is a counterpoise box filled with lead.

H. Driving clock: the frictional governors revolve on ball bearings, and the friction comes into play horizontally on a ring of polished tool steel.



ELEVATION OF 30-INCH REFLECTOR.

An Apparent Influence of the Earth on the Numbers and Areas of Sun-spots in the Cycle 1889–1901.

By A. S. D. Maunder.

(*Communicated by Sir W. H. M. Christie, K.C.B., Astronomer Royal.*)

1. *Introductory.*

The material used in the following paper is derived from the Photo-heliographic Results published by the Astronomer Royal in the Greenwich Observations from the years 1889–1901 inclusive. During these years the results of the measures made at Greenwich of photographs of the Sun, taken daily at Greenwich, in India, and in Mauritius, are exhibited in two chief forms. In the first, the measures of positions and areas of sun-spots are given day by day. In the second, they are arranged in the form of "Ledgers" of the groups of spots, and it is these ledgers which I have utilised for the present inquiry. These ledgers of the spot groups were first given for the year 1886, but as a complete cycle of spot activity ran its course during the years 1889–1901, I have confined my present investigation to those thirteen years. The Astronomer Royal, in his last Report to the Board of Visitors, has announced that he is having similar ledgers prepared for the years 1874–1885, and when these appear I hope to treat the cycle 1878–1889 in the same way as I have here treated the cycle which followed it.

The object of my inquiry is to ascertain whether these measures of the solar photographs afford any support to the idea that the Earth exercises, or seems to exercise, any perceptible influence, either on the numbers or on the areas of the spot groups.

The following is a sample of one of the sun-spot ledgers:—

TABLE I.

Extract from Ledger of Sun-spots.

Date. Greenwich Civil Time.	Projected Area of		Area for Group.		Mean Longitude of Group.	Mean Latitude of Group.	Longitude from Central Meridian.
	Umbra.	Whole Spot.	Umbra.	Whole Spot.			
Group 5104.							
A very fine stream following Group 5103. The stream is so compact that it is frequently measured as consisting chiefly of one very large composite spot. By November 13, two very large spots, *a* and *b*, have formed at the front and rear of the group respectively.							
Nov. 4·515	17	164	59	561	168·9	−23·8	−79·0
5·452	64	458	105	752	166·8	−24·1	−68·7
6·531	92	764	96	809	164·8	−24·3	−56·6
7·466	109	1150	88	925	164·3	−24·2	−44·8
8·225	124	1274	86	879	162·8	−24·5	−36·2
9·526	210	1471	125	882	163·3	−24·4	−18·6
10·265	213	1349	122	775	163·0	−24·5	− 9·1
11·194	193	1356	109	771	163·8	−24·9	+ 4·0
12·146	233	1389	139	828	163·1	−25·3	+15·7
13·195	192	1163	126	767	163·4	−25·1	+29·9
14·561	78	682	70	588	163·9	−25·0	+48·4
15·154	96	463	102	483	164·2	−25·0	+56·5
16·228	47	266	88	462	163·7	−24·8	+70·2
17·298	3	37	12	132	159·7	−25·0	+80·2
Means ...	...	...	95	687	163·98	−24·64	...

The several columns of the ledger are described in the Introduction to the Greenwich Observations.

Spots, brought into view by the rotation of the Sun on its axis, are first seen at the east limb, and pass out of view at the west limb; the longitudes from the central meridian given in the eighth column being reckoned as negative when the spots are east of the central meridian and positive when they are west.

In the following discussion, the data of which I make use are those given in the fifth and eighth columns respectively, that is to say, the areas (expressed in millionths of the Sun's visible hemisphere, and corrected for foreshortening) of the whole spots, and the longitudes of the groups as reckoned from the central meridian of the Sun.

The period of rotation assumed for the Sun in the Greenwich Photo-heliographic Results is 25·38 days. This is, of course, the siderial period of rotation; the corresponding mean synodic rotation is 27·275 days, and the mean apparent daily motion of a spot with respect to the central meridian will be 13°·2. A long-lived spot will appear to cross the visible disc of the Sun in thirteen or fourteen days. I have therefore, for the purpose of this investigation, divided the Sun's visible hemisphere into fourteen lunes, of which seven are east of the central meridian and seven are west, each lune being 13°·2 of longitude in breadth. In the lune corresponding to the "Longitude from the Central Meridian" for each day of apparition of a spot group, I have entered the area of the spot group, corrected for the effect of foreshortening, for that day. For example, the spot group already cited would be entered as on Table II.

2. *Preponderance of Areas of Sun-spots in the Eastern Hemisphere.**

In all, 2870 spot groups have been thus treated, the total number of entries being 15,721, for few of the groups were observed for fourteen days, the average duration being about six days. Table III. gives the result of summing up the total areas for each lune, first, for the northern hemisphere, and then for the southern, and thirdly for the entire disc of the Sun.

It will be seen that in both hemispheres there is a regular progression in the spotted area from the first lune to the seventh, and a corresponding decrease from the eighth lune to the fourteenth.

The areas in the first and in the fourteenth lunes are much smaller than those in the remaining twelve. The reason for this is obvious. A spot is practically invisible when it is distant 86° from the central meridian. The effective breadth, therefore, of the first and last lunes is only half that of any of the remaining twelve. It must also be borne in mind that spot groups in general extend over several degrees of longitude, so that the centre of a group may

* Strictly speaking, this is only a quarter of the sphere, but it seems unnecessarily pedantic to coin a new term.

Table II.

Extract from the Form for entering the Areas in the Lunes.

Date when first seen.	No. of Group.	1st Lune. −79°·3 and over.	2nd Lune. −66°·1 to −79°·2	3rd Lune. −52°·9 to −66°·0	4th Lune. −39°·7 to −52°·8	5th Lune. −26°·5 to −39°·6	6th Lune. −13°·3 to −26°·4	7th Lune. − 0°·1 to −13°·2	8th Lune. + 0°·1 to +13°·2	9th Lune. +13°·3 to +26°·4	10th Luue. +26°·5 to +39°·6	11th Lune. +39°·7 to +52° 8	12th Lune. +52°·9 to +66°·0	13th Lune. +66°·1 to +79°·2	14th Lune. +79°·3 and over.	Total Area of Group.	Mean Hel. Long. of Group.	Mean Hel. Lat. of Group.
1903. 11.4	5104	561	752	809	925	879	882	775	771	828	767	588	483	462	132	9614	164°	−25°

Table III.

Lunes.

Eastern Hemisphere. Western Hemisphere.	1st −79°·3 and over. 14th +79°·3 and over.	2nd −66°·1 to −79°·2 13th +66°·1 to +79°·2	3rd −52°·9 to −66° 0 12th +52°·9 to +66°·0	4th −39°·7 to −52°·8 11th +39°·7 to +52°·8	5th −26°·5 to −39°·6 10th +26°·5 to +39°·6	6th −13°·3 to −26°·4 9th +13°·3 to +26°·4	7th − 0°·1 to −13°·2 8th + 0°·1 to +13°·2	Eastern Hemisphere. Western Hemisphere.
Northern Spots.								
East.	24358	74826	89979	99224	102708	105138	106216	602449
West.	21302	71476	87947	93799	100949	103883	104679	584035
Difference E. − W.	+3056	+3350	+2032	+5425	+1759	+1255	+1537	+18414
Southern Spots.								
East.	35622	95195	111741	119763	125225	128292	128995	744833
West.	27370	90480	108816	117282	124169	128585	129869	726571
Difference E. − W.	+8252	+4715	+2925	+2481	+1056	−293	−874	+18262
All Spots.								
East.	59980	170021	201720	218987	227933	233430	235211	1347282
West.	48672	161956	196763	211081	225118	232468	234548	1310606
Difference E. − W.	+11308	+8065	+4957	+7906	+2815	+962	+663	+36676

fall within the first or last lune, though much of its area may be actually beyond the visible hemisphere. The areas in these two lunes ought therefore to be multiplied by some factor greater than 2, to make them really comparable with the other twelve.

But omitting these two lunes, we yet see a very marked progression in the other twelve; the areas steadily increase from the east to the central meridian, and diminish from the central meridian to the west.

What can be the cause of this? Two suggestions immediately present themselves. There may be some systematic error in the measurement, or there may be some systematic error in the reduction.

There are several causes which can be conceived as possibly giving rise to an error of the second kind. First of all the intense brightness of the Sun may, by irradiation, make the photographic radius systematically appear greater than it really is, with the result that the distance of the spot from the centre of the disc, expressed in terms of the radius, will be too small. An effect in the same direction would also arise from the measurer judging the centre of the spot as if he were dealing with a marking on a plane surface. This again would tend to put the position of the centre of the spot slightly too near the centre of the disc, since the effect of foreshortening will always be greatest on the side of the spot furthest from the centre. Yet again, if there is any sensible displacement of the position of a spot due to the effect of refraction in the solar atmosphere, this would also make the spot appear nearer the centre of the disc than it really is. The effect of each of these hypothetical sources of error in the measurement would be the same; the distance of a spot from the centre would be under-estimated, and too small a correction would be applied for foreshortening, and as the factor increases with the sine of the angular distance from the centre, the areas of the lunes near the limb would require to be increased in a higher proportion than those near the centre.

If the steady progression from the east limb to the central meridian, and from the central meridian to the west limb, were the only feature shown by Table III., it would be necessary to examine very carefully in detail each of the above hypothetical sources of error, trivial though they certainly must be. But there is another feature, just as clearly shown, into which any error in the correction for foreshortening cannot enter. If we compare the area of the eastern hemisphere with the area of the western, we find that the eastern is always the greater, whether we consider the north or the south. And further, if we compare each of the several lunes in the eastern hemisphere with the corresponding lune in the western, we find that, almost without exception, the eastern is the greater, usually much the greater.

This difference between the two hemispheres cannot be due to any error of foreshortening, seeing that the hemispheres and their constituent lunes are symmetrical. Neither can it be due to any personal equation in the measurement of the solar photographs;

the form of the micrometer and the method of measuring preclude that. (See Introduction to Greenwich Observations.)

The amount of the predominance of the areas in the eastern hemisphere over those in the western is too marked, both for the hemispheres as a whole, and for their constituent lunes, for it to be accidental, whatever explanation may be assigned for it.

3. *Preponderance of Numbers of Spot Groups in the Eastern Hemisphere.*

There is a simple way of getting rid of any systematic error in the measurement or reduction of the areas of the spots; *i.e.*, we may discard the areas and deal only with the number of groups. This is what has been done in Tables V. and VI. following. It will be observed that not only are the total numbers of groups counted in each lune given, but the groups are arranged in classes. All spots do not live for the same length of time. They may come into existence on the invisible hemisphere of the Sun, or on the visible hemisphere; and similarly, they may die out either on the invisible or on the visible hemisphere. They may be seen for one, two, or any number of consecutive days up to fourteen. If very long-lived, they may be so seen in a second, third, fourth, or even in a fifth rotation, but in this discussion I have treated each apparition of a spot group as a separate and independent display, except where the contrary is expressly stated. Regarding, therefore, fourteen days as the longest period for which a spot group can be followed consecutively, I have thought it well to divide the whole cycle of the sun-spots of 1889–1901 not only into those in the northern and southern hemispheres of the Sun respectively, but also into classes distinguished by the number of days during which the group was visible; that is to say, into classes of spots observed for fourteen consecutive days, for thirteen, for twelve, and so on, down to ephemeral groups which were observed only upon a single day. On the average, I found that the mean area of a group bore a direct proportion to the length of time for which it was visible. The following Table IV. gives the number of groups which existed for each of the given periods, the total area of the groups, and the mean area per group. Here it may be noted that both the number of groups and the mean area per group are slightly greater in the southern than in the northern hemisphere.

In Table V. the third section of Table IV. is dealt with, but the totals for each lune in the northern and southern hemispheres separately are added at the foot. The spots are not only divided into classes according to the length of time that they were under observation, but each lune is treated separately, and the number of groups observed in that particular lune is recorded. The total numbers observed in the eastern and western hemispheres are likewise exhibited.

From the nature of the case, the number of spots of the first class, namely, those observed for fourteen days, is the same for all

Table IV.

Total Areas and Mean Areas of Groups of Different Durations.

No. of Days.	Northern Spots.			Southern Spots.			All Spots.		
	No. of Groups.	Total Area.	Mean Area.	No. of Groups.	Total Area.	Mean Area.	No. of Groups.	Total Area.	Mean Area.
			Per Day.			Per Day.			Per Day.
14	10	55893	399	21	184142	626	31	240035	553
13	100	455948	351	99	477671	371	199	933619	361
12	93	291688	261	108	343318	265	201	635006	264
11	57	116564	186	66	129800	179	123	246364	182
10	54	65709	122	69	93245	135	123	158954	130
9	62	55491	99	61	53414	97	123	108905	98
8	61	45504	93	84	56715	84	145	102219	88
7	55	30602	80	73	41870	82	128	72472	81
6	65	17564	45	80	28300	59	145	45864	53
5	73	20919	57	85	17319	41	158	38238	48
4	88	12358	35	133	23228	44	221	35586	40
3	98	7724	26	113	11560	34	211	19284	30
2	135	6336	23	168	6169	18	303	12505	21
1	349	4184	12	410	4653	11	759	8837	12
Total,	1300	1186484	165	1570	1471404	172	2870	2657888	169

Table V.

Number of Groups in each Lune for each Class of Duration of Spot Groups.

No. of Days.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	E.	W.	E.–W.
14	31	31	31	31	31	31	31	31	31	31	31	31	31	31	217	217	0
13	109	199	199	199	199	199	199	199	199	199	199	199	199	90	1303	1284	+ 19
12	33	186	201	201	201	201	201	201	201	201	201	201	168	15	1224	1188	+ 36
11	18	82	116	123	123	123	123	123	123	123	123	105	41	7	708	645	+ 63
10	16	63	82	112	123	123	123	123	123	123	107	60	41	11	642	588	+ 54
9	12	52	70	93	114	123	123	123	123	111	71	53	30	9	587	520	+ 67
8	15	67	86	96	110	137	145	145	130	78	59	49	35	8	656	504	+152
7	8	40	61	64	77	94	124	120	88	67	64	51	34	4	468	428	+ 40
6	3	39	64	81	88	96	110	102	81	64	57	49	32	4	481	389	+ 92
5	9	42	59	68	84	84	66	67	82	74	65	50	32	8	412	378	+ 34
4	9	47	77	104	107	83	76	69	71	83	68	48	34	8	503	381	+122
3	2	33	58	71	53	40	52	58	63	55	59	50	31	8	309	324	− 15
2	0	30	59	51	43	56	66	54	49	52	50	55	36	5	305	301	+ 4
1	1	35	64	66	65	73	94	94	83	64	50	37	28	5	398	361	+ 37
Total,	266	946	1227	1360	1418	1463	1533	1509	1447	1325	1204	1038	772	213	8213	7508	+705
North,	124	428	563	625	636	661	697	682	655	617	560	474	349	103	3734	3440	+294
South,	142	518	664	735	782	802	836	827	792	708	644	564	423	110	4479	4068	+411

lunes. For the second class, those observed for thirteen days, the number is the same for each lune except the first and last. Similarly for the successive classes, until we come to spots observed for less than half a complete transit, that is to say, for seven days and under. For the spots of long duration, therefore, the number of spots observed in each lune near the centre of the disc must necessarily be constant. But when we exclude these, it will be noted that not only do the numbers in the whole of the eastern quadrant exceed those in the whole of the western in nearly every case, but in most instances the number in any particular eastern lune exceeds that in the corresponding western lune.

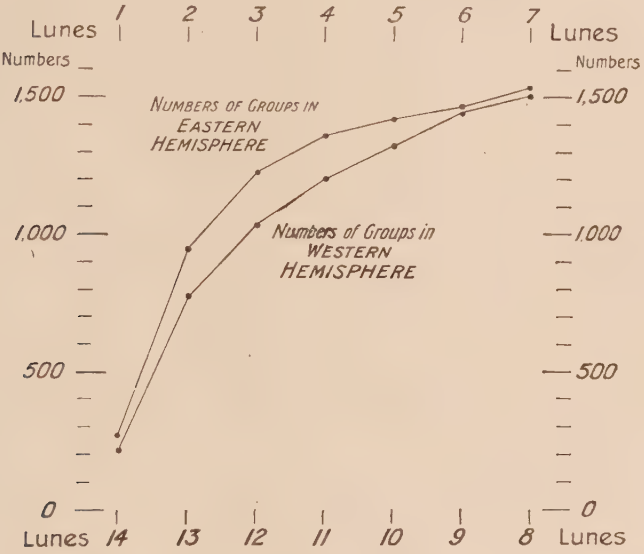


Fig. 1.—Numbers of Groups in each Lune, comparing East and West.

Table VI. (represented graphically in fig. 1) gives the differences in the number of groups between the corresponding eastern and western limbs, regardless of the class of the spot, that is to say, of the length of time that it was under observation. It will be seen that both for the northern spots and for the southern, and consequently for the two combined, the eastern lune always shows an excess over the corresponding western lune as to the number of spots observed. In no single case does the number of groups in a western lune equal those in the corresponding eastern one. The preponderance of east over west attains its maximum in the third lune from the limb, where the proportion is almost exactly 7 to 6.

TABLE VI.

Differences in Numbers of Groups found in Corresponding Lunes, E.–W., for Northern, Southern, and All Spots.

	1–14	2–13	3–12	4–11	5–10	6–9	7–8	E.–W.
Northern,	+21	+ 79	+ 89	+ 65	+19	+ 6	+15	+294
Southern,	+32	+ 95	+100	+ 91	+74	+10	+ 9	+411
All Spots,	+53	+174	+189	+156	+93	+16	+24	+705

4. *Preponderance of Numbers of Spot Groups rising in the Unseen Hemisphere.*

It has already been shown that the eastern hemisphere has a systematic preponderance over the western hemisphere in respect both to total spotted area and the numbers of spot groups. The question naturally arises, Can we ascertain whether there is any difference between the visible and invisible hemispheres in respect to the numbers of groups, and, if so, which hemisphere has the advantage?

Something can be learnt from the comparison of the numbers of groups seen in the two lunes nearest to the east limb on the one side, with those given by the two lunes nearest to the west limb. For, from the nature of the case, these numbers most nearly represent the number of groups which come round into view from the unseen hemisphere on the one side, and pass away into it on the other.

TABLE VII.

Numbers of Groups seen in the First or Second Lune, and in the Thirteenth or Fourteenth Lune, with their differences, for Northern, Southern, and all Spots.

No. of Days.	North.			South.			Whole Sun's Disc.		
	1st or 2nd Lune.	13th or 14th Lune.	Diff. E.–W.	1st or 2nd Lune.	13th or 14th Lune.	Diff. E.–W.	1st or 2nd Lune.	13th or 14th Lune.	Diff. E.–W.
14	10	10	0	21	21	0	31	31	0
13	100	100	0	99	99	0	199	199	0
12	85	79	+ 6	101	89	+12	186	168	+ 18
12	34	23	+11	48	18	+30	82	41	+ 41
10	33	12	+21	30	29	+ 1	63	41	+ 22
9	27	13	+14	25	17	+ 8	52	30	+ 22
8	26	14	+12	41	21	+20	67	35	+ 32
7	20	13	+ 7	20	21	− 1	40	34	+ 6
6	16	15	+ 1	23	17	+ 6	39	32	+ 7
5	21	18	+ 3	21	14	+ 7	42	32	+ 10
4	22	14	+ 8	25	20	+ 5	47	34	+ 13
3	16	12	+ 4	17	19	− 2	33	31	+ 2
2	9	16	− 7	21	20	+ 1	30	36	− 6
1	10	13	− 3	26	20	+ 6	36	33	+ 3
Total,	429	352	+77	518	425	+93	947	777	+170

This comparison is brought out in Table VII. where, as in Tables IV and V., the numbers are given for northern and southern spots

separately and combined, and for all classes of spots. With very few exceptions, the number of groups that come into view out of the hemisphere that is remote from the Earth exceeds that of the groups that pass out of sight from our Earth into the unseen hemisphere. It is further significant that the exceptions are almost entirely confined to spots of very short duration, not exceeding three days.

Now this persistent preponderance cannot be a mere chance effect; the signs in the third columns are too constantly positive for anything that is purely accidental. Yet the eastern and western lunes have no significance at all, when considered from a standpoint on the *Sun*; their distinctiveness lies in their different relations to the *Earth*. Putting the matter shortly, we see that, in all, 947 spot groups came round the eastern limb of the Sun into view of the Earth, and only 777 passed round from the view of the Earth at the western limb into the invisible hemisphere. And whether we consider the Sun as a whole, or its northern and southern hemispheres separately; or whether we consider the spot groups with or without regard to their length of life, we get a result to the same effect. We are driven to conclude that during the cycle, 1889–1901, the Earth was apparently responsible for the extinction of about 170 spot groups; that is to say, of more than one-sixth of the whole number that came into view round the eastern limb. Whether this effect was produced by bringing about the premature dissolution of old groups, or by hindering the generation of new groups, or by a combination of the two, there is no doubt as to the magnitude of the resulting effect.

5. *Birth-rate and Death-rate of Spot Groups in the Different Lunes.*

It will be interesting to examine whether there is any tendency for spots to take their origin in one lune rather than another, and also whether certain lunes are especially favourable for their dissolution. It is manifest that the fact that a spot is first seen in lunes 1 and 2 affords very small evidence that it actually arose in those lunes; the strong probability is that it arose in the unseen hemisphere. Similarly, the fact that a spot is last seen in either lune 13 or 14, affords no presumption that its actual dissolution took place there. But when we are dealing with the ten central lunes we have a certain amount of positive evidence that the groups first seen or last seen in these regions of the disc were not visible earlier or later. The effect of foreshortening will no doubt be to hide a few of the smaller groups, whilst in the two lunes nearest to either limb, and in a few quite accidental cases, this may even extend to the third lune from each limb; but the record of appearances and disappearances for at least the central lunes must be substantially correct.

Table VIII., which may be considered as a supplement to Table VII., gives the number of groups first seen in each lune from the 3rd to the 12th inclusive, and also the number of those last seen.

Ephemeral spots, that is those only seen on a single day, are not included in this table.

TABLE VIII.

Numbers of Groups first seen or last seen in the Ten Central Lunes.

Lune.	First Seen.			Last Seen.		
	North.	South.	All Spots.	North.	South.	All Spots.
3	113	139	252	10	22	32
4	72	91	163	33	36	69
5	50	78	128	39	55	94
6	54	77	131	45	57	102
7	70	81	151	62	74	136
8	47	65	112	61	75	136
9	41	44	85	66	116	182
10	39	40	79	80	82	162
11	27	28	55	100	92	192
12	17	22	39	116	146	262
Total,	530	665	1195	612	755	1367

Since the 3rd lune is symmetrical with the 12th, the 4th with the 11th, and so on, any error, from such effects of foreshortening, in the number of spots first seen in the 3rd lune, will be of the same amount as that in the number of spots last seen in the 12th lune; and so with the other pairs of corresponding lunes. The difference, therefore, ought to be free from any such effect of foreshortening, and to represent not an apparent effect but a real difference.

Table IX. is derived from Table VIII., and exhibits the differences in the number of groups first seen in any particular lune, with the number last seen in the corresponding lune.

TABLE IX.

Comparison of Numbers of Groups first seen in any Lune with those last seen in the Corresponding Lune.

Seen first in Lune.	Seen last in Lune.	North.	South.	All Spots.
3	12	− 3	− 7	− 10
4	11	− 28	− 1	− 29
5	10	− 30	− 4	− 34
6	9	− 12	− 39	− 51
7	8	+ 9	+ 6	+ 15
8	7	− 15	− 9	− 24
9	6	− 4	− 13	− 17
10	5	− 0	− 15	− 15
11	4	− 6	− 8	− 14
12	3	+ 7	− 0	+ 7
Totals,		− 82	− 90	− 172

From Table IX., therefore, we see that there is an average excess per lune of 17 dissolutions over formations; this being made up of 8 in the northern hemisphere and 9 in the southern, which is very nearly in the proportion of the total number of groups of either hemisphere. For every 7 groups that form on the visible disc, 8 dissolve, or at least one-seventh more groups form on the invisible hemisphere than on the visible. The death-rate on the visible hemisphere is systematically higher than the birth-rate.

When we deal with the distribution of the ephemeral spots (see Table V.), we find a steady increase in number per lune from the east to the central meridian, and a decrease thence to the west limb. But the ratio of those generated on the eastern side to those generated on the western is about 11 to 10, being an excess of 37 in a total number of 755.

In Table III. a comparison was made between the total spotted area shown by the eastern and western hemispheres, the area of the former being 1347282 and of the latter 1310606 millionths of the Sun's visible hemisphere; the excess of the eastern hemisphere was thus 2·8 per cent. In Table VII., in which the number of groups in the two most easterly lunes were compared with those in the two most westerly, the excess of the easterly amounted to 21·9 per cent. In Table V., where the groups seen in each lune were summed up together, the preponderance of the eastern hemisphere was 9·4 per cent. In Table V. the ephemeral groups show a preponderance in the eastern hemisphere of 10·3 per cent. over the western; whilst the number of dissolutions given in Table VIII. shows a predominance over the formations of 14·3 per cent. It will be seen that, though the predominance of the eastern hemisphere over the western in the matter of area is well marked and consistent, yet that it is by no means so great in proportion as is the predominance in the number of groups. This would appear to indicate that, whatever was the cause of the predominance of the eastern hemisphere, it was one which affected small groups to a greater degree than large ones. But if we group the areas in the same way as we have already grouped the numbers, we shall see the introduction of another factor which will render the last inference more clear.

6. "*Spot Phase.*"

In Tables X. and XI. the total areas are given for each lune, but the spots of the different classes are exhibited separately. Both tables are for the whole disc of the Sun; but in Table XI. and in fig. 2 the areas are expressed, not in millionths of the Sun's visible hemisphere, but each class of spot has been taken separately, and the area of each lune has been expressed in thousandths of the total area for that class. These present the remarkable feature that the maximum area for any class of spot tends to shift over from the east towards the west as we go from the spots of long duration to those of short.

TABLE X.*

Areas for each Lune for All Spots of Different Durations.

Total.

No. of Days.	1	2	3	4		6	7	8	9	10	11	12	13	14	East.	West.
14	10381	18467	20883	21621	21781	21302	19590	19180	17880	16592	16705	15560	12628	7465	134025	106610
13	31116	65321	74909	81254	82086	81353	80459	77608	76474	73221	66456	61456	53382	22061	496498	430658
12	7699	43536	53858	56202	56298	57587	58412	57634	56075	53523	49667	44913	34832	4770	333592	301414
11	3367	12654	15503	18508	22917	25568	25599	24859	24167	22255	18438	16013	10858	1607	124116	118197
10	2007	7142	8652	11634	13786	15441	16412	16487	16214	14597	13549	11611	9414	2008	75074	83880
9	1429	5936	6538	7984	9917	11488	11627	10768	9952	9990	8049	7272	5523	2254	54919	53808
8	1898	6169	7024	6985	6942	7558	7962	10046	10638	10494	9878	8837	6428	1360	44538	57681
7	946	2927	3596	3650	3905	4047	5264	7278	8491	8900	8638	8022	5826	719	24335	47874
6	163	2112	2630	2845	2629	2155	2362	3189	4889	5415	5514	6178	5150	486	14896	30821
5	469	1584	2125	1994	1835	1654	1773	2323	2816	3496	4646	5756	5609	1841	11434	26487
4	417	1844	2234	2562	2311	1731	1769	1466	1533	2988	4688	5149	5097	1797	12868	22718
3	64	984	1469	1278	1089	722	956	1008	1032	1151	2075	3199	3042	1071	6562	12578
2	0	592	888	791	569	813	823	858	543	861	1071	1368	2787	624	4476	8112
1	13	427	719	727	610	737	916	727	681	673	611	583	909	503	4149	4687

TABLE XI.

Expressed in Thousandths.

14	43	77	87	90	91	89	82	80	74	69	70	65	53	31	558	442
13	34	70	81	88	89	88	87	84	82	79	72	66	58	24	536	464
12	12	69	85	88	89	91	92	91	88	84	78	71	55	8	525	475
11	14	52	64	76	95	106	106	103	100	92	76	66	45	7	512	488
10	13	45	54	73	87	97	103	104	102	92	85	73	59	13	472	528
9	13	54	60	73	91	105	107	99	91	92	74	67	51	21	505	495
8	19	61	69	68	68	74	78	99	104	103	97	87	63	13	435	565
7	13	41	50	51	54	56	73	101	118	123	120	111	81	10	337	663
6	4	46	58	62	58	47	52	70	107	118	121	135	113	11	326	674
5	12	42	56	53	48	44	47	61	74	92	122	152	148	49	301	699
4	12	52	63	72	65	49	50	41	43	84	132	145	143	51	362	638
3	3	52	77	67	57	48	50	53	54	60	109	167	159	56	343	657
2	0	47	71	63	45	65	65	68	43	68	85	109	221	50	356	644
1	3	48	81	82	69	83	104	82	77	76	69	66	103	57	470	530
Total,	23	64	76	82	86	88	88	88	87	85	79	74	61	18	507	493

* There is a slight difference in the material employed in Tables III. and X. Table III. embraces all spots of the cycle 1889–1901, whilst Table X. does not extend beyond the end of 1900.

The curves given by the 14- and 13-day spots are somewhat flat, and come to a maximum on the fifth day. The curve continues to be very flat for the 12-day spots, but the maximum is shifted to the seventh lune. The 11-day spots show a more decided curve, and by the time the 8-day spots are reached the maximum is

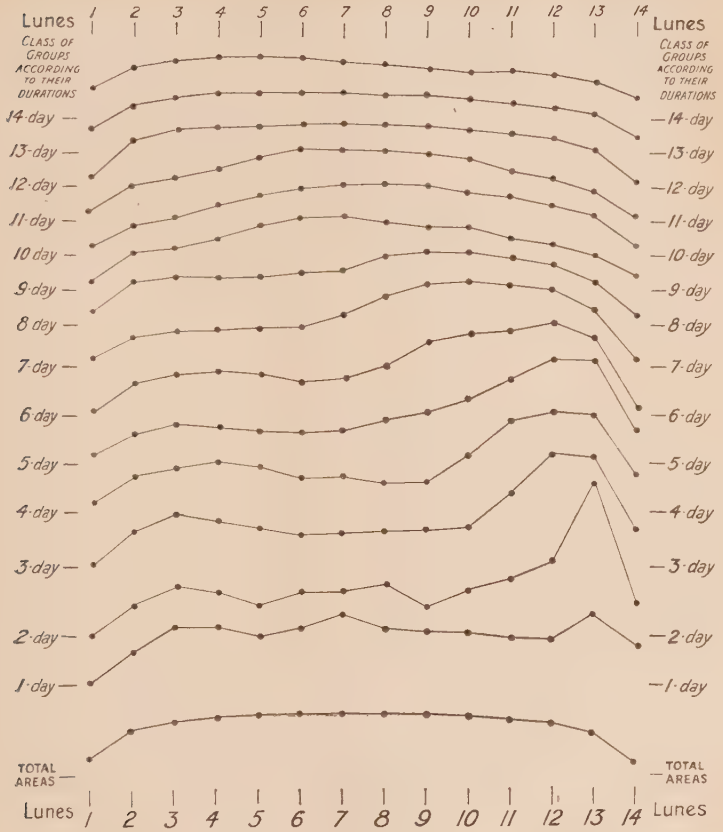


FIG. 2.—Curves of the Variation in Area for Spots of different Durations as seen in the different Lunes.
(The areas are expressed in thousandths of the total area for each class.)

found in the ninth lune. The 6-day spots show two maxima, a small one in the fourth lune and a much greater in the twelfth. The spots of 5-, 4-, and 3-day duration give curves of the same form, but with the two maxima more and more accentuated. The 2-day groups show a tremendous peak in the thirteenth lune, beside two small maxima, the one in the third lune and the other at the centre of the disc. The ephemeral spots also have three maxima,

one in the fourth lune, one in the seventh, and one in the thirteenth. The total areas of all spots, irrespective of duration, give very nearly a smooth curve, but the eastern lunes systematically show a slight preponderance over the western.

It is thus seen that the preponderance in area of the eastern hemisphere over the western, as shown in Table III., is one which is entirely confined to the classes of spots which have long duration. Table XII. brings this out yet more clearly, for it exhibits in tabular form the differences between the corresponding lunes east and west; the areas for each class being expressed as in Table XI., in thousandths of the total area of that class. It shows that in the long-lived groups there is a steady preponderance of the east over the west, not only for the two hemispheres of the Sun, but also for each pair of lunes. It shows also how, contrariwise, for the short-lived groups, the west preponderates still more emphatically over the east. Just the same result is obtained if the northern and southern hemispheres are treated separately, though the figures naturally show greater irregularities, since the numbers treated are smaller.

TABLE XII.

Differences between the Corresponding Lunes, E.–W., expressed in thousandths of Total Area for Whole Sun.

No. of Days.	1–14	2–13	3–12	4–11	5–10	6–9	7–8	E.–W.
14	+12	+ 24	+22	+20	+22	+15	+ 2	+116
13	+10	+ 12	+15	+16	+10	+ 6	+ 3	+ 72
12	+ 4	+ 14	+14	+10	+ 5	+ 3	+ 1	+ 50
11	+ 7	+ 7	− 2	+ 0	+ 3	+ 6	+ 3	+ 24
10	− 0	− 14	−19	−12	− 5	− 5	− 1	− 56
9	− 8	+ 3	− 7	− 1	− 1	+14	+ 8	+ 10
8	+ 6	− 2	−18	−29	−35	−30	−21	−130
7	+ 3	− 40	−61	−69	−69	−62	−28	−326
6	− 7	− 67	−77	−59	−60	−60	−18	−348
5	−37	−106	−96	−69	−44	−30	−14	−398
4	−39	− 91	−82	−60	−19	+ 6	+ 9	276
3	−53	−107	−90	−42	− 3	−16	− 3	−314
2	−50	−174	−38	−22	−23	+22	− 3	−288
1	−54	− 55	+15	+13	− 7	+ 6	+22	− 60
Total,	+ 5	+ 3	+ 2	+ 3	+ 1	+ 1	+ 0	+ 14

What, then, can be the cause of this striking difference in the behaviour of groups of long duration and of short duration? Why should the former show their greatest areas in the eastern hemisphere, and the latter so markedly in the western?

The explanation is very simple. Three circumstances combine to bring it about.

First, whilst groups observed for twelve, thirteen, or fourteen days are undoubtedly groups of long duration, groups observed for only three or four days are by no means necessarily of short duration. They may have existed in the unseen hemisphere before they came under observation, or they may have continued to exist there after they had passed beyond the range of observation. The time during which we observed them may in many cases represent simply the beginning or the close of the life-history of long-duration groups, so that the spots classed as of short duration must really include a great number of long duration.

Next, it has been already pointed out in Table IV., that the longer the life-history of a spot, the greater on the average will be its mean area.

Third, just exactly as the spotted area of the Sun increases much more quickly as the cycle runs up from minimum to maximum than it decreases when the cycle runs down from maximum to minimum, so is it with the great majority of individual spot groups—the growth in area is more rapid than is the decline. This characteristic we may denote by the term "spot phase."

7. *Analysis of the Spot Groups of 8-days apparent Duration.*

It follows, therefore, if we are dealing with one of the classes of short-duration spots, say, for example, with those of eight days, that beside the true 8-day spots, we have a number of spots really of long duration but classed as short because they formed in the invisible hemisphere, and only the last eight days of their existence were passed on this side of the Sun. Whilst these groups are under observation they are decaying, and their chief energy is already passed, so that their areas are not of a much higher order than those of true 8-day groups. The areas, therefore, in the eastern hemisphere will be but little affected. But it will not be so in the western hemisphere. Beside the true 8-day groups there will also be a number of long-duration groups just forming, and classed with them because observed for eight days before they passed to the other side of the Sun. Being spots of long duration they will on the average be of great mean area, and being observed during the period of rapid growth, their areas will be of quite a different order from those of the true 8-day groups with which they have been classified.

Tables XIII. and XIV. bring out this effect very clearly. In Table XIII., the 8-day groups, whether they genuinely existed for only eight days or simply appeared to do so, have been divided into three categories:—Eastern groups, that is to say, groups seen first close to the east limb, *i.e.* in the first or second lunes; Central groups, first seen in the third, fourth, or fifth lunes, and last seen in the tenth, eleventh, or twelfth lunes, thus clearly having their origin and dissolution in the visible hemisphere; and

Western groups, those last seen close to the west limb, *i.e.* in the thirteenth or fourteenth lunes. The first category will, therefore, be largely made up of groups which formed in the unseen hemisphere, but dissipated on the visible disc. The last category will

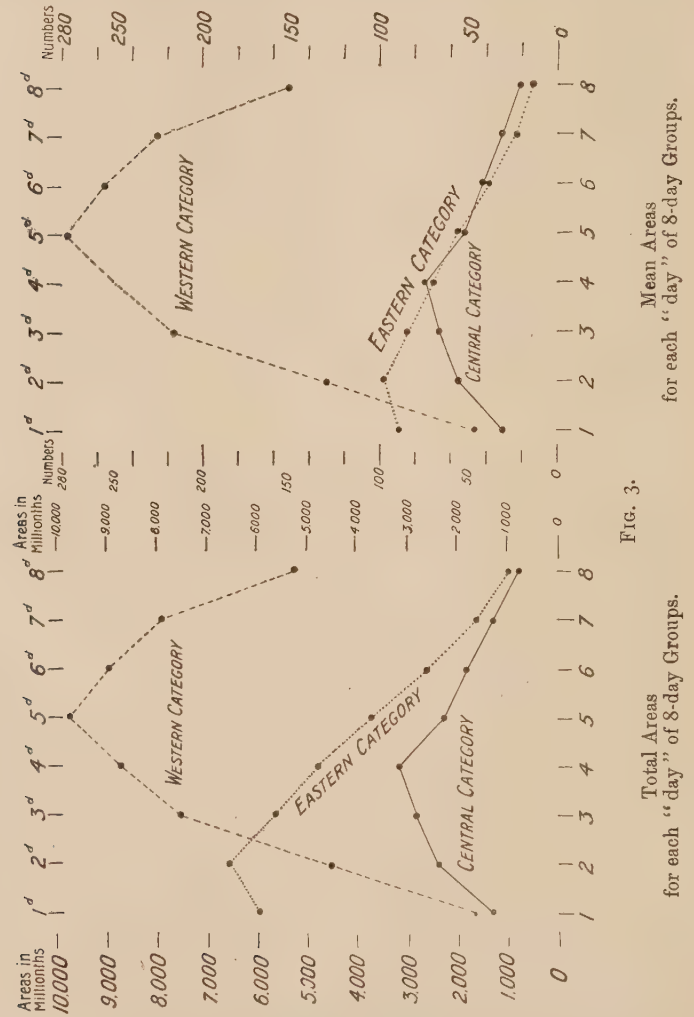


FIG. 3.

include many which, though they formed on the visible hemisphere, did not dissipate until after they had passed out of view at the west limb. Many therefore of both these classes are not true 8-day groups; they only appeared to be so, but really had a longer duration, and this would mean that on the average they were groups of greater area than true 8-day groups.

Table XIII.

Total Areas for each Day of Life of Eastern, Western, and Central Groups of the 8-Day Spots, for Northern, Southern, and All Spots.

Class of Group.	1st Day.	2nd Day.	3rd Day.	4th Day.	5th Day.	6th Day.	7th Day.	8th Day.	Number of Groups.
				Northern.					
Eastern,	2587	2637	2085	1845	1409	1124	671	444	26
Western,	967	2593	3921	3916	4058	3606	3183	2832	14
Central,	725	1437	1296	1439	970	768	600	379	21
				Southern.					
Eastern,	3348	3908	3541	2933	2292	1495	941	541	41
Western,	691	1916	3635	4803	5666	5337	4723	2460	21
Central,	597	956	1531	1712	1309	1049	715	527	22
				All Spots.					
Eastern,	5935	6545	5626	4778	3701	2619	1612	985	67
Western,	1658	4509	7556	8719	9724	8943	7906	5292	35
Central,	1322	2393	2827	3151	2279	1817	1315	906	43

In the first place, it should be noted that the number of groups in the Eastern category is almost double the number of groups in the Western, and yet the total area in the Eastern is much less than in the Western, so that the mean area of an Eastern group is very much less than the mean area of a Western. These mean areas are given in Table XIV. Fig. 3 presents both Tables XIII. and XIV. in graphical form.

Table XIV.

Mean Area for each Day of Life of Eastern, Western, and Central Groups of the 8-Day Groups, for Northern, Southern, and All Spots.

Class of Group.	1st Day.	2nd Day.	3rd Day.	4th Day.	5th Day.	6th Day.	7th Day.	8th Day.
				Northern.				
Eastern,	100	101	80	71	54	43	26	17
Western,	69	185	280	280	290	258	227	202
Central,	35	68	62	69	46	37	29	18
				Southern.				
Eastern,	82	95	86	72	56	34	23	13
Western,	33	91	173	229	270	254	225	117
Central,	27	44	70	78	60	48	33	24
				All Spots.				
Eastern,	89	98	84	71	55	39	24	15
Western,	47	129	216	249	278	256	226	151
Central,	31	56	66	73	53	42	31	21

The distinction between the "lunes," into which the visible disc of the Sun is divided, and the "days" in the life-history of a spot group, should be clearly borne in mind. The first *day* of the life of an Eastern group may fall in the first or in the second *lune*; its eighth *day* will fall in the eighth or ninth *lune*. The first *day* of a Central group may fall in the third, in the fourth, or in the fifth *lune*. The first *day* of a Western group may fall in the sixth or in the seventh *lune*. But in Tables XIII. and XIV. the areas for each day of the life of a group, in the several categories, are summed irrespective of the lunes in which they may actually fall; and since the first and the fourteenth lunes have really but half the area of any of the remaining twelve lunes, the areas of the *first day* of the Eastern category, and of the *eighth day* of the Western category, should be each multiplied by a factor greater than unity.

There is a very marked difference, not only in the total and mean areas of the three categories, but also in their several progressions. The Eastern category has a steady decay from the second day of its life—or indeed from its first day, if we allow for the fact that the first lune is smaller in area than other lunes,—and the eastern "day" has a greater area than the corresponding western "day"; the fourth day is greater than the fifth, the third than the sixth, etc. The Western category increases in area until its fifth day, after which it again steadily declines; and here it is the western "day" that is always greater than the corresponding eastern. In the Central category, the culmination is on the fourth day, and again the areas for the eastern "days" are greater than for the corresponding western.

And, considering the three categories, we find very strong evidence of the action of "spot phase." The groups of the Central category may take their rise over a range of the third, fourth, and fifth lunes; and as on the whole disc of the Sun there are 43 such groups, there is an average of 14 groups arising in each lune, the mean area of each group being 47. We may assume that this is the average number of true 8-day groups, arising in each lune all over the Sun, and that true 8-day groups average 47 in area. But we find, on comparing these numbers and mean area with those of the Eastern and Western categories, though these have but a range of one and a half lunes (half, that is to say, the range of the Central category), that these latter rank much greater both in numbers and mean areas. In both the Eastern and Western categories the true 8-day groups are swamped by the greater numbers and areas of the longer-lived groups classed as 8-day groups, because they are only *observed* for eight days in the visible hemisphere; the true 8-day groups seem to exercise little perceptible modification of the course run by either the Eastern or the Western category of groups.

The Eastern category must then be composed—overwhelmingly, both in numbers and areas—of the last eight days of decaying groups, and these final eight days show a steady decline. The Western category must be composed—overwhelmingly in areas, if not in the numbers of the groups—of the first eight days of active

and long-lived groups, and must include in the mean their period of greatest activity, for the mean area culminates on the fifth day of their apparent life. Comparing the Eastern and Western categories, we find that the Eastern has very nearly double the number of spot groups of the Western, but that the Western has nearly double the total area and very nearly four times the mean area of the Eastern.

This detailed examination of the 8-day groups, therefore, renders two things clear. First, that for groups of this class—observed for eight days—many more come round at the east limb from the unseen hemisphere and die on the visible disc, than rise on the visible hemisphere and pass out of sight at the west limb.

Next, we have a clear explanation of the gradual shift of the maxima towards the west, as shown in Tables X. and XI. It is simply a function of "spot phase."

It is not possible to treat in the same manner groups that have a longer visible duration than 10 days. We cannot discriminate between those groups which existed for only from 12 to 14 days, and those which rose or dispersed in the unseen hemisphere. We should reasonably expect the former to be but a small fraction of the latter; but for them the influence of "spot phase" would be to throw the maximum somewhat to the east of the central meridian, since the rise of spot groups is more rapid on the average than their decline. But for groups that are observed as of shorter duration than 10 days, the curves in fig. 2 show the influence of the three categories of Eastern, Central, and Western. Already in the 6-day class, the preponderance in area and numbers of the Eastern category over the Central has produced a secondary maximum in the fourth lune, the greater maximum due to the Western falling in the twelfth. For the remaining classes, we see the effect of the Eastern categories in the swift run up in the early lunes, and subsequent decline. In the 2-day and 1-day spots, the central maximum is without doubt due to the better presentation there which enables small spots to be seen which might have been passed over altogether when foreshortened.

8. *Preponderance as to Area of the Eastern Hemisphere in Long-lived and Recurrent Groups.*

But there is a class of groups which must be free from any tendency of "spot phase" to link their greatest development with any particular lune. If we consider only those groups which have been seen in three or more distinct apparitions, and, rejecting the first and last, take only the intermediate appearances, then we are clearly dealing with the most stable portion of the life-history of the most permanent and important groups. We are not confusing together short-lived and unimportant groups with those which are greater and more durable, but are accidentally under observation for only a short time. Nor are we in any way introducing the condition that they must either take their rise or dissipate in any particular lune. In Table XV., therefore, I give for the northern hemisphere

the total areas in each lune of the intermediate appearances of the groups which have been under observation, in three or more distinct rotations, the first and last appearances not being used. The total areas are also exhibited in the same table, for each lune, for all the 13- and 14-day groups in the northern hemisphere. It will be seen that there is an unfailing predominance of the eastern hemisphere

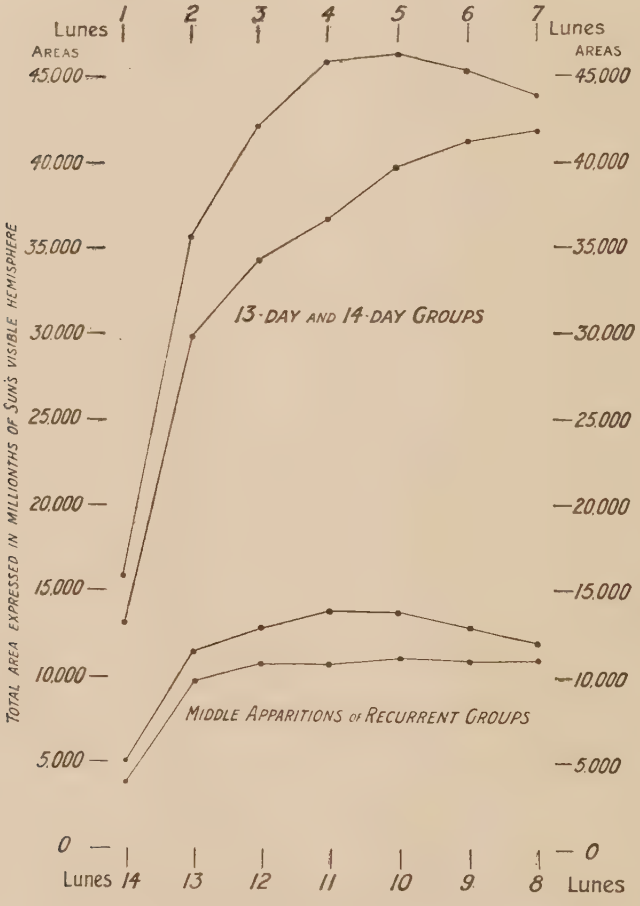


FIG. 4.—Total Areas for each Lune of long-duration and recurrent Groups.

over the western, but that the recurrent groups give a more regular curve than the 13- and 14-day groups. A comparison of the two sets of figures shows that there is a slight heaping up of areas in the central lunes for the 13- and 14-day groups as compared with the recurrent groups, which is due no doubt to the influence of "spot phase." But nevertheless, in both cases the predominance of the

eastern areas over the western culminates in the central lune of each hemisphere. Fig. 4 represents Table XV. graphically.

TABLE XV.

Areas in each Lune of Middle Apparitions of Recurrent Groups in Northern Hemisphere.

Lunes.	1	2	3	4	5	6	7	East.
	14	13	12	11	10	9	8	West.
	5035	11401	12665	13800	13660	12934	12047	81542
	3824	9847	10815	10734	11092	11022	10953	68287
Diff. E. – W.	+1211	+1554	+1850	+3066	+2568	+1912	+1094	+13255

Areas in each Lune for all 13-Day and 14-Day Groups in Northern Hemisphere from Table VII.

	15677	35601	42280	45949	46129	45325	43888	274849
	13052	29913	34277	36616	39894	41311	41929	236992
Diff. E. – W.	+2625	+5688	+8003	+9333	+6235	+4014	+1959	+37857

9. *Summary.*

The foregoing tables show that there is a well-marked and steady preponderance of the eastern half of the Sun's disc over the western half, both as to the areas of the spots and as to the numbers of the separate groups. This excess in area of the eastern hemisphere over the western amounts to 17 per cent. for spots observed consecutively for 13 or 14 days; to more than 19 per cent. for recurrent spots during their time of greatest stability; and to 3 per cent. for the totality of areas of spots of all sizes and durations. As to the numbers of groups, the excess of the eastern hemisphere is 10 per cent. for ephemeral spots; is more than 9 per cent. if the groups are summed up as seen in the different lunes; and is 22 per cent. if we compare the groups seen in the two lunes nearest to the east limb with those seen in the two lunes nearest to the west limb. This last relation is the most striking which the inquiry has brought out. No fewer than 947 groups came into view round the east limb, or formed close to it, whilst only 777 disappeared round the west limb, or dissolved close to it. So that the difference of 170, or 22 per cent. of the number of disappearances, must be laid to the account of some influence exercised by the Earth. And these disproportions either in area or in number cannot be put down to any cause connected with the history or growth of the spots themselves, or to any solar cause whatever. East is east, and west is west, solely from an earthly point of view. From a solar standpoint, the "east limb," "central meridian," and "west limb" are purely conventional lines behind which every portion of the Sun's surface moves; they are not fixed landmarks upon the Sun. The

cause of the disproportion must then be terrestrial, and terrestrial only.

It must be terrestrial only, but it may also be apparent only, being due in some way or other to an effect of presentation.

Some error must creep in from the effect of foreshortening. Foreshortening pure and simple is, of course, corrected for in the ordinary process of reduction, but it is probable that a few minute corrections may be further required on grounds already alluded to. Further, some spots, large enough to be seen and measured when near the centre of the disc, will wholly escape notice if they happen to be near the limb. Thus foreshortening may affect to some slight degree not only the areas, but also the numbers of the spot groups.

But, from the nature of the case, the effect of foreshortening must be symmetrical, with respect to the central meridian, and no systematic difference between the eastern and western hemispheres can arise from this cause. "Spot phase" (as has been already shown) has a distinct asymmetrical effect, when the spots of any particular class are considered by themselves; but when the totality of all spots is considered, this asymmetrical effect must go out, since (unless we admit that the Earth has a real effect upon the Sun) it is not possible that any particular phase in the life-history of a spot should be associated with any particular meridian of the Sun as seen from the Earth.

It does not appear, therefore, that it can be a mere effect of presentation.

I have called this paper, therefore, "An *Apparent* Influence of the Earth on the Numbers and Areas of Sun-spots," because it is quite conceivable that there may be some physical characteristic of sun-spots, purely solar in its nature, which renders them more easily visible when they are approaching us than when they are receding. Thus if we suppose that sun-spots are holes in the photosphere of which the axes are generally sloped downward from west to east, or if we suppose that there is a great heaping up of the surface behind a sun-spot, we might have some explanation of the predominance of the eastern hemisphere both in the numbers and areas of sun-spots. Neither of these suggestions would, in my opinion, account for the relations which I have put in evidence, and there is another class of phenomenon which seems to show the same predominance of the eastern hemisphere over the western which could hardly be affected in the same way.

10. *Preponderance of the Numbers of Prominences in the Eastern Hemisphere.*

I had completed the foregoing statistics when it occurred to me to ascertain whether prominences were more frequently observed on the east or on the west limb, and in making this supplemental inquiry I came across a note by M. Sykora in the *Memorie della Società degli Spettroscopisti Italiani* for 1897, p. 161, in which he drew attention to the circumstance that there was a

slight but unmistakable predominance of eastern prominences. I carried on the summation of the prominences as given from the Catania Observatory in Sicily to the end of 1905, and found that from 1892–1900 inclusive the eastern hemisphere was the more prolific in every year. The tables were turned, however, with the year of minimum, 1901, and for four years the west was slightly the more favoured. In 1905 the east again predominated. It

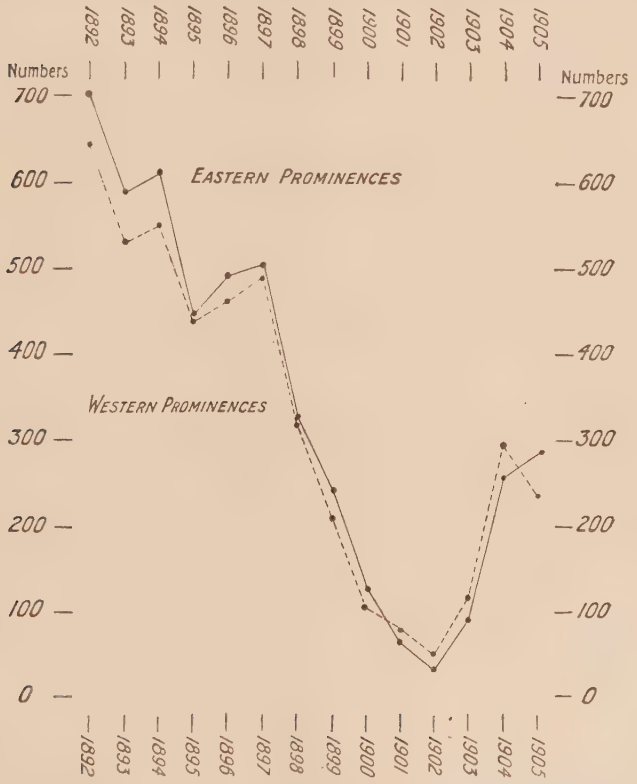


Fig. 5.—Numbers of Prominences as observed in different Years on the East and West limbs of the Sun at the Royal Observatory, Catania.

will be seen, therefore, that for the period with which my paper is concerned, namely, the cycle 1889–1901, the prominences showed the same feature as the sun-spots; and it seems very difficult to suggest any function of presentation, either connected with the solar constitution or with the conditions of observation, which would affect both phenomena in the same way. Table XVI. and fig. 5 exhibit the comparison of the numbers of prominences observed on the east and west limbs at the Catania Observatory.

TABLE XVI.

Prominences observed at Catania in Sicily.

Year.	E.	W.	Total.	E.-W.
1892	700	641	1341	+ 59
1893	589	530	1119	+ 59
1894	613	552	1165	+ 61
1895	446	439	885	+ 7
1896	491	462	953	+ 29
1897	504	493	997	+ 11
1898	324	319	643	+ 5
1899	242	213	455	+ 29
1900	134	107	241	+ 27
1901	64	80	144	− 16
1902	33	49	82	− 16
1903	91	118	209	− 27
1904	256	295	551	− 39
1905	282	238	520	+ 44
Total,	4769	4536	9305	+ 233

The results to which I have come in the foregoing paper are not what I expected to find when I began this inquiry. Indeed, they are the very opposite of what has been supposed would be the result of planetary action, if it had any effect on the solar disturbances. It has been usually assumed that if a planet had any influence upon the Sun, it would be in the nature of spot production, not spot extinction. But the evidence of the foregoing tables seems to show that spots tend to diminish in area rather than to increase as they pass under the Earth, and that there is a decided tendency to check the generation of spots on the hemisphere presented to the Earth.

11. *Preponderance in Spotted Area at Apogee.*

I have no speculations to offer as to the nature of this terrestrial influence upon the solar spot groups. Were it in any way analogous to a tidal effect, we might expect to find some slight difference in the mean spotted area of the Sun in perigee and in apogee. In Table XVII., therefore, I have given the mean daily spotted area per month for two periods of eleven years each, namely, 1880–1890 and 1891–1901, and also for the twenty-two years taken together. The area in each month of any year has been expressed in thousandths of the sum of the twelve monthly mean areas for that year. On the whole, the summer months, when the Earth is most distant from the Sun, show the larger spotted area; May to August giving a mean of 86, whilst November to February give a mean of 79. There are, however, big

irregularities, as the comparatively short period of twenty-two years is insufficient to eliminate accidental irregularities.

TABLE XVII.

Mean Daily Spotted Area per Month, expressed in thousandths of the Total Area, for the Periods 1880–1890, 1891–1901, *and* 1880–1901.

Period.	Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
1880–1890	74	76	78	95	84	89	93	73	89	89	93	67
1891–1901	87	93	67	75	84	88	93	95	96	78	62	82
1880–1901	81	85	72	84	84	89	93	85	93	83	76	75

This table I drew up several years ago, but it seemed to me so contrary to what one would naturally expect, that the spotted area should be greatest at apogee, that I did not then pursue the inquiry further. But in the light of the statistics that I have given in this present paper, it suggests that the inquiry is well worth pursuing with regard to the other planets.

The Earth is of such insignificant size, not merely as compared with the Sun, but even as compared with very many individual sun-spots, that its influence (if any such exists) on their number and size might well have been expected to be imperceptible. I commenced this investigation fully expecting to obtain a purely negative result; I was certainly not prepared to find that the effects of the Earth's apparent influence (of whatever nature this may be), would be so great, so striking, so consistent, and seen in so many directions.

69 *Tyrwhitt Road, St John's, Brockley, S.E.:*
1907 *May* 6.

Appendix added 1907 *May* 28.

It has been suggested to me that I might supplement the above tables by some statistics as to the numbers of groups that are born or die on the visible or invisible hemisphere respectively. This is done in Table XVIII., in which each group has been counted only once, no matter in how many rotations it has been seen. In other words, the numbers have been corrected for the recurrence of long-lived groups.

TABLE XVIII.

Born and dying on Visible Hemisphere.

	N.	S.	Whole Disc.
Short-lived groups	700	829	1529
Groups seen in more than one apparition .	39	55	94
Total . .	739	884	1623

TABLE XVIII.—*continued.*

	N.	S.	Whole Disc.
Born on Visible and dying on Invisible Hemisphere.			
Groups seen only in one apparition . .	107	129	236
Groups seen in more than one apparition .	25	39	64
Total . .	132	168	300
Born on Invisible and dying on Visible Hemisphere.			
Groups seen only in one apparition . .	166	225	391
Groups seen in more than one apparition .	43	36	79
Total . .	209	261	470
Born and dying on Invisible Hemisphere.			
Groups seen only in one apparition . .	28	31	59
Groups seen in more than one apparition .	21	22	43
Total . .	49	53	102
Grand Total . .	1129	1366	2495
Groups seen twice	128	152	280
Groups seen three or more times . . .	43	52	95
Total . .	1300	1570	2870

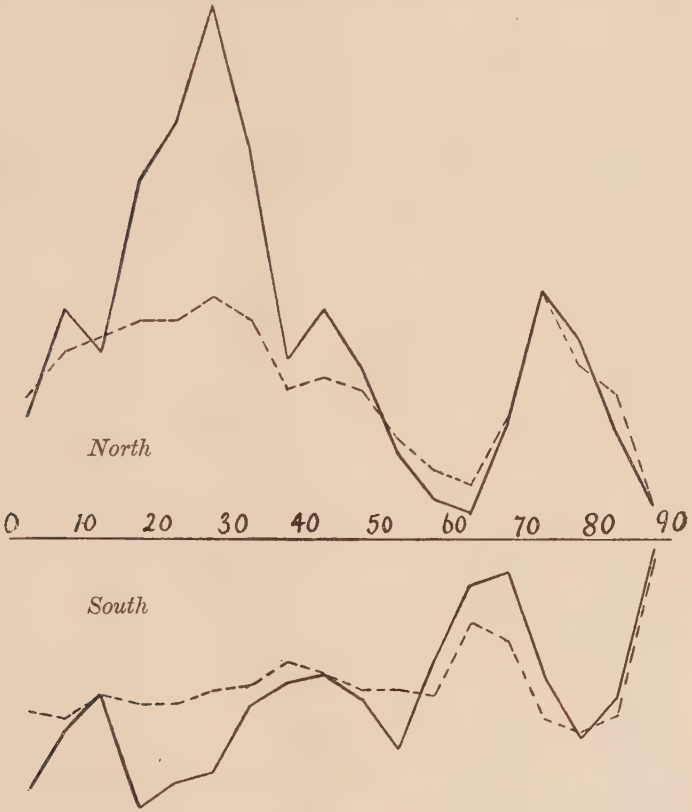
Neglecting therefore the short-lived spots that are born and die on the visible hemisphere of the Sun, we find

TABLE XIX.

	N.	S.	Whole Disc.
Born on visible hemisphere	171	223	394
,, invisible ,,	258	314	572
Dying on visible ,,	248	316	564
,, invisible ,,	181	221	402

Distribution of Prominences in Latitude in the Year 1906, *from observations made at Kodaikánal on* 156 *days in the first half of the year, and* 105 *days in the second half.* By J. Evershed.

The distribution of prominences in latitude for each half of the year 1906 is represented in the diagrams by two curves. The



Distribution of prominences in latitude—1906 January 1 to June 30.

Broken line = mean numbers
Continuous line = mean activity
} for each 5 degrees.

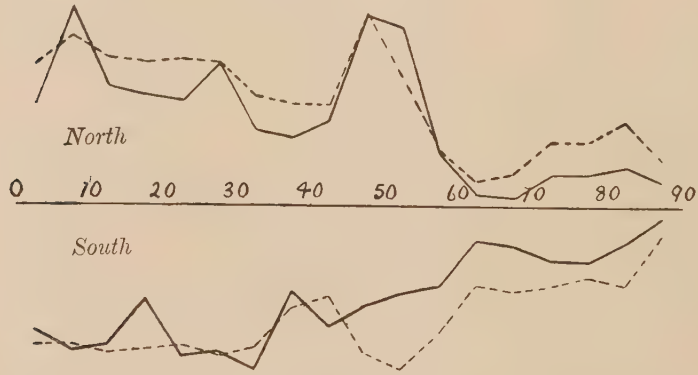
broken line gives the mean numbers of prominences observed in each zone of 5° from the equator to the poles. The continuous line gives the mean "activity" for the same zones. The activity is obtained by taking account of the height and extent of each prominence, the unit adopted being a prominence of 1° in extent and 10″ in height.

The average size of the prominences in each zone is indicated by the difference of the ordinates of the two curves: thus, at latitudes where the continuous line rises above the broken line in the northern hemisphere (or falls below it in the southern), the prominences were larger than the average for the whole year, and at the crossing points they were of average size.

General Remarks.

The distribution curve for the first half of the year has the same form as that of the previous year, with a maximum in the zone 25°–30°, and a well-defined zone of activity in high latitudes.

On both sides of the equator there is a very pronounced minimum of activity in the zones 55°–65°.



Distribution of prominences in latitude—1906 July 1 to December 31.

Broken line = mean numbers
Continuous line = mean activity
} for each 5 degrees.

About the end of June a considerable change took place in the distribution, and in the second half of the year the curve takes a very different form. There is also a great falling off in activity in both hemispheres.

The year as a whole is interesting, as showing the culminating point in the prominence period, when the high-latitude zones of activity finally envelope the poles. This took place last at about the epoch 1894·7. At that time, as in the past year, the zones of maximum activity were at latitudes 25°–30° on both sides of the equator, and during the past 12 years these regions of greatest activity have been slowly advancing towards the poles. In the years 1896 to 1901 they remained practically stationary between 45° and 55°, but since that date the advance polewards has been at the average rate of 5° per annum. New maxima appeared

between latitudes 20° and 30° in 1902, and the shifting of the old maxima towards the poles appears to be correlated with the development of these.

An interesting feature in these high-latitude prominences is the narrow limits of latitude within which at any one time the zone is defined, and the tendency to form extended chains of prominences all approximately in the same latitude, but often exceeding 100° in longitude. When this occurs one may get an apparently stationary prominence, remaining at the same position angle for 10 or 14 days, after which it appears on the opposite side of the pole for another series of days.

Observations of Jupiter's Sixth and Seventh Satellites from Photographs taken with the 30-inch Reflector at the Royal Observatory, Greenwich, in 1906–7.

(*Communicated by the Astronomer Royal.*)

The Sixth and Seventh satellites of Jupiter have been under observation at Greenwich from 1906 August 28 to 1907 April 6, and photographs have been taken at every available opportunity. In all, 55 photographs of J. VI. have been obtained on 28 nights, the observations extending over a period of 222 days, and 12 photographs of J. VII. on 7 nights extending over a period of 87 days. All these have been measured and reduced and the results are given in the present paper.

In addition to the satellites, eight reference stars, whose positions were taken from the Astronomische Gesellschaft Catalogue (Berlin B. Zone), were measured on each photograph and the constants determined in the usual manner. Right ascensions and declinations of the satellites were then determined, and by comparison with the tabular place of Jupiter, position angles and distances deduced.

Observations of Satellite VI.

Date and G.M.T. 1906.				Apparent R.A.	Apparent Dec.	Position Angle.	Distance.	Exp.
d	h	m	s	h m s	° ′ ″	°	″	m
Aug. 28	15	23	29	6 22 42·60	+22 37 48·1	208·053	1712·8	28
31	15	24	32	6 24 58·69	+22 36 25·1	203·733	1672·8	45
Sept. 25	13	43	0	6 40 52·04	+22 26 21·9	158·992	1662·5	40
25	14	28	50	6 40 53·01	+22 26 21·4	158·935	1662·5	40
26	14	16	1	6 41 23·20	+22 26 5·7	157·130	1678·1	41
Oct. 13	14	2	11	6 48 14·10	+22 24 21·6	129·782	2137·5	54
16	13	44	50	6 49 3·42	+22 24 45·1	125·897	2245·7	20
16	14	35	51	6 49 3·74	+22 24 44·9	125·910	2244·9	53
19	16	28	9	6 49 46·76	+22 25 23·0	122·183	2363·6	30

Observations of Satellite VI.—continued.

Date and G.M.T. 1906.					Apparent R.A.			Apparent Dec.			Position Angle.	Distance.	Exp.
	d	h	m	s	h	m	s	°	′	″	°	″	m
Oct.	27	12	59	35(*a*)	6	50	58·60	+22	28	14·3	113·838	2670·0	13
	27	13	36	28	6	50	58·76	+22	28	14·8	113·817	2671·2	40
	27	14	38	59	6	50	58·90	+22	28	15·6	113·793	2671·8	60
	27	15	49	32	6	50	59·17	+22	28	16·6	113·755	2673·9	57
Nov.	17	11	3	37	6	49	29·92	+22	44	44·1	96·457	3336·5	30
	17	11	59	59	6	49	29·40	+22	44	46·7	96·425	3337·8	57
	22	12	44	17	6	48	4·59	+22	50	30·5	92·685	3409·5	20
	22	13	17	17	6	48	4·23	+22	50	32·8	92·655	3410·8	25
Dec.	6	9	20	42	6	42	9·44	+23	8	18·9	81·815	3294·6	30
	6	9	54	21	6	42	8·75	+23	8	21·1	81·793	3294·9	20
	6	11	3	46	6	42	7·21	+23	8	24·6	81·755	3293·2	10
	10	11	15	35(*b*)	6	39	53·79	+23	13	43·5	78·103	3159·1	20
	10	11	47	22	6	39	53·08	+23	13	45·6	78·080	3159·2	15
	10	12	45	49	6	39	51·69	+23	13	49·6	78·028	3157·9	80
	12	11	32	38	6	38	42·36	+23	16	21·6	76·103	3074·8	60
	12	12	27	24	6	38	40·96	+23	16	26·0	76·035	3073·1	30
	12	12	57	9	6	38	40·25	+23	16	27·7	76·017	3072·9	10
	12	13	13	32	6	38	39·82	+23	16	28·9	75·997	3072·0	10
	14	10	17	2	6	37	30·57	+23	18	53·6	74·009	2982·9	21
	14	10	50	10	6	37	29·66	+23	18	54·7	73·992	2980·9	31
	21	11	6	37	6	32	52·53	+23	27	20·9	64·962	2564·9	30
	21	11	31	13	6	32	51·81	+23	27	22·3	64·927	2563·7	15
	1907.												
Jan.	17	7	20	27	6	13	56·30	+23	43	16·7	329·545	1525·4	15
	17	8	36	17	6	13	54·33	+23	43	16·7	329·351	1527·2	120
	17	12	4	12	6	13	48·78	+23	43	16·3	328·750	1533·2	44
	17	12	56	35	6	13	47·42	+23	43	16·4	328·620	1534·6	32
	22	8	18	23(*c*)	6	10	50·63	+23	42	50·0	311·757	1799·9	30
	22	8	55	6	6	10	49·70	+23	42	49·0	311·657	1801·6	30
	22	9	33	19	6	10	48·78	+23	42	48·4	311·570	1803·0	15
Feb.	1	10	40	41(*d*)	6	5	41·74	+23	39	30·8	290·539	2444·0	15
	1	11	0	38	6	5	41·43	+23	39	29·9	290·511	2444·1	10
	2	10	6	30(*e*)	6	5	16·97	+23	39	5·0	289·135	2501·9	16
	7	7	41	50	6	3	27·65	+23	36	34·0	282·900	2757·6	60
	7	8	55	56	6	3	26·65	+23	36	31·4	282·825	2759·6	60
	7	9	44	6	6	3	25·93	+23	36	31·5	282·807	2762·2	5
	7	9	51	34	6	3	25·87	+23	36	30 3	282·782	2761·7	7

(*a*) Faint and diffused.
(*b*) Very faint. On the réseau line.
(*c*) Bad image; a little out of focus.
(*d*) Very faint.
(*e*) Nearly coincident with a faint star.
(*f*) Very diffused.

Observations of Satellite VI.—continued.

Date and G.M.T. 1907.				Apparent R.A.			Apparent Dec.			Position Angle.	Distance.	Exp.
	d	h	m s	h	m	s	°	′	″	°	″	m
Feb.	8	10	15 13	6	3	6·36	+23	35	56·1	281·637	2809·0	120
	8	11	37 27(*d*)	6	3	5·36	+23	35	56·3	281·620	2811·1	15
	11	9	10 1	6	2	16·20	+23	34	17·5	278·632	2929·8	86
	15	10	6 39(*d*)	6	1	22·76	+23	31	57·9	274·947	3064·0	24
Mar.	4	8	6 52(*f*)	6	0	48·89	+23	22	33·6	262·460	3238·6	120
	4	9	36 14	6	0	49·25	+23	22	31·6	262·417	3239·1	30
	10	9	55 30	6	1	47·39	+23	19	37·3	258·460	3181·2	106
	11	8	33 1	6	1	59·59	+23	19	9·0	257·800	3168·5	147
	11	10	22 0(*d*)	6	2	0·62	+23	19	9·2	257·792	3166·9	30
	18	8	57 3	6	3	55·59	+23	16	14·2	253·305	3042·7	65
Apr.	6	8	19 43	6	12	28·97	+23	9	30·2	239·872	2528·5	60

(*d*) Very faint.
(*f*) Very diffused.

Observations of Satellite VII.

Date and G.M.T. 1906.				Apparent R.A.			Apparent Dec.			Position Angle.	Distance.	Exp.
	d	h	m s	h	m	s	°	′	″	°	″	m
Nov.	17	11	59 59	6	47	58·03	+22	53	51·8	85·408	2057·2	57
	22	12	44 17	6	46	6·94	+22	59	28·8	78·242	1815·5	20
	22	13	17 17(*a*)	6	46	6·29	+22	59	31·4	78·165	1813·3	25
Dec.	12	11	32 38	6	35	32·88	+23	24	12·6	17·282	1257·2	60
	12	12	27 24(*b*)	6	35	31·48	+23	24	15·7	17·150	1258·1	30
	1907.											
Jan.	17	8	36 17	6	11	58·73	+23	55	37·9	311·093	3134·5	120
	17	12	4 12	6	11	53·68	+23	55	43·1	311·010	3143·7	44
	17	12	56 35(*b*)	6	11	52·39	+23	55	42·8	310·965	3145·2	32
Feb.	7	7	41 50	6	2	44·02	+24	0	14·6	301·897	3860·6	60
	7	8	55 56(*c*)	6	2	43·04	+24	0	17·1	301·903	3863·8	60
	8	10	15 13	6	2	25·27	+24	0	15·0	301·542	3878·8	120
	11	9	10 1	6	1	41·17	+24	0	13·5	300·695	3915·7	86

(*a*) Faint and diffused.
(*b*) Faint.
(*c*) Almost coincident with a faint star.

In deducing the position angle and distance given above, the error of the tabular place of Jupiter has been neglected.

To determine this error a number of photographs of Jupiter were taken with the 26-inch Refractor, using the occulting shutter. By this means good measureable images of Jupiter were obtained, together with sufficiently exposed images of the reference stars. The field of the 26-inch, on a 16 c.m. plate, is one square degree, and all the stars in this area given in the Astronomische

Gesellschaft Catalogue were measured together with Jupiter, and the position of Jupiter deduced with reference to them.

It will be observed that the deduced place of Jupiter will be affected by the systematic error of the Catalogue, and the error due to unknown proper motions, and for that reason it is directly comparable with the positions of the satellites deduced in the same manner.

Five photographs were selected for measurement, and the results are given below.

Errors of Tabular Place (T − O).

1907.	R.A.	Dec.
	s	"
Feb. 28	− ·06	+0·2
Mar. 1	− ·06	+0·4
1	− ·05	−0·2
20	− ·05	−0·7
21	− ·07	+0·1
Mean	− ·058	−0·04

The photograph on March 20 is unsatisfactory.

The tabular place is taken from the *Nautical Almanac*.

Royal Observatory, Greenwich:
1907 *May* 10.

Note on the Spectrum of α Orionis. By H. F. Newall and B. Cookson.

Titanium Flutings in the Red End of the Spectrum of α Orionis.

A photographic study of the red end of the spectrum of some of the brighter stars has been made during the past two months at the Cambridge Observatory with a four-prism spectrograph attached to the 25-inch refractor. It has led to the discovery, amongst other things, of a marked feature in that part of the spectrum of *α Orionis*, viz. three strong flutings, which in this note are interpreted as absorption flutings, with heads towards the blue end. Only a preliminary determination of the wave-lengths of these heads has been made; for, on account of the paucity of lines in the spark spectrum of iron, which has been used as a comparison spectrum, some uncertainty—to the extent of about 2 tenth metres—is attached to the identification of a red line near 7155, which has been used as one of the reference lines.

This line 7155 and the stellar flutings are in truth outside of the range of spectrum, for which the four-prism spectrograph had

been adjusted, and the definition is not what it ought to be for a careful determination of wave-length.

However, the interest of the matter lies in the facts (i) that the provisional values deduced for the wave-lengths of the heads of the flutings are—

7053
7087
7124

and (ii) that Hale and Adams have in their paper (*Astroph. Jour.*, xxv., 77) given the wave-lengths of three absorption flutings which they have found in the spectrum of Sun-spots, and have identified with bright flutings found by them in the spectrum of the flame of the titanium arc, with heads at wave-lengths—

7054·6
7088·0
7125·9

Thus a chain of evidence seems to be completed :—

(*a*) Professor Fowler discovers many flutings in the flame of the titanium arc and points out the close resemblance, in position and intensity, between these bright flutings and the absorption flutings seen in stellar spectra of type III. (like *α Orionis*).—(*Proc. R.S.* lxxiii. 219.)

(*b*) Messrs Hale and Adams have proved the close resemblance between those features which distinguish both Sun-spot spectra and the spectrum of *α Orionis* from the solar spectrum, viz. certain lines which are present in both Sun-spots and *α Orionis*, with intensity accentuated relatively to the corresponding solar lines (*Astroph. Journ.*, xxiii. 400). (Some further evidence on this point is given in the later paragraphs of the present note; it tends to support the contention of Messrs Hale and Adams.)

(*c*) Messrs Hale and Adams, in their latest note (*Astroph. Jour.*, xxx. 77) on Sun-spot spectra, announce that they have been able to identify flutings in the Sun-spot spectrum as reversals of the bright flutings at the red end of the spectrum of the flame of the titanium arc.

(*d*) The object of the present note is to announce that these dark flutings have been detected in photographs of the red end of *α Orionis*.

Attempts have been made to detect these red flutings in *β Geminorum* and *α Boötis*; but the material so far collected is not quite good enough to establish their presence or absence with complete certainty.

Sun-spot Lines in the Green Region of the Spectrum of α Orionis.

Three photographs of the green region of the spectrum of *α Orionis* were taken by Mr H. J. Bellamy in January 1905 with a four-prism spectrograph. They have been recently measured by

Mr W. H. Manning, and the measures have been discussed with a view to seeing whether the lines which were relatively stronger in *α Orionis* than in the Sun were Sun-spot lines. The regions covered are as follows :—

Plate No.	Date 1905.	Limits of Spectrum studied.
Fmg. 680	Jan. 13	5098–5328
Fmg. 684	Jan. 14	5108–5324
Fmg. 696	Jan. 27	5341–5521

Within these regions many dark (absorption) lines were measured, and also some apparently bright (emission) lines, though it was difficult to say whether these latter were really bright lines or only bright interstices of continuous background between dark absorption lines.

The wave-lengths of all the lines measured are given in the table below. They have been corrected for velocity: up to and including 5328·28 the tabulated values are in general the means of the two plates Fmg. 680 and Fmg. 684; the rest are from Fmg. 696. The scale of the original photographs is 1 mm. = 25 Ångström units.

Photographs of this region of the spectrum of the Sun and of Sun-spots were taken last summer, and it was thus possible to compare the star lines with solar lines and see whether they were lines that were intensified in spots. The results of comparison with three different photographs of Sun-spot spectra (Nos. R.S. 187, 188, 189) are given in the table. A cross (×) indicates that the line was relatively stronger in the spot than in the Sun; o that it was not; ? that intensification in the spot was doubtful. Unfortunately there is a gap in our series of photographs of Sun-spot spectra between 5341 and 5600.

A list of Sun-spot lines was published in *Astroph. Jour.*, xxiii. 15–27, by Hale and Adams, and the fourth column shows those lines which are included in their list, and are considered by them as lines affected in spots.

This table shows clearly that a large number of the star lines are also spot lines. The following lines are almost certainly not spot lines:

All the bright lines marked B.
Lines 5133·61, 5145·44, 5273·21.

The so-called bright lines cannot be identified for certain: it was thought they might be reversals of the Sun-spot fluting of which the wave-lengths are given in *M.N.*, Dec. 1906, but this view has not been upheld. It seems probable that they are not really bright lines.

The other three are not well-marked lines:

5133·61 is very faint and hazy.
5145·44 is fairly strong but very hazy on the red side.
5273·21 is extremely faint.

The last four lines in the table are not very well seen on the

Wave-Lengths of Lines in α Orionis.

α Orionis λ in star.	Sun-spot Lines			α Orionis λ in star.	Hale and Adams.
	R.S. 187. / R.S. 188.	R.S. 189.	Hale and Adams.		
5099·10 *	×		o	5341·17	×
5107·72	×	×	o	5371·80	×
5110·56	?	×	×	5395·01	×
5123·71	?	×	×	5397·23	×
5127·75	o	×	?	5406·04	×
5133·61 *	o	o	o	5409·94	×
5135·66 *B				5429·92	×
5137·20	o	o	×	5434·91	o
5138·23 B				5447·08	×
5139·44 *	?	?	×	5455·95	o
5140·86 *B				5490·92	×
5143·03	×	?	o	5497·84	×
5144·32 B				5501·61	o
5145·44	?	?	o	5506·83	×
5147·82	×	×	×	5512·49	
5169·12 *	?	?	×	5514·68	×
5190·29 *B				5517·51	
5203·62	o	×	o	5520·97	
5208·59	×	?	×		
5210·52	×	×	×		
5219·93	×	×	o		
5227·30	?	×	×		
5228·93 B					
5230·25	×	×	×		
5231·64 *B					
5247·31	×	×	×		
5250·40	o	×	×		
5255·27	×	×	×		
5264·22	×	×	×		
5265·21 *	o	×	×		
5266·28	×	?	×		
5267·91 B					
5269·92	?	?	×		
5273·21 *	?	?	o		
5275·56 *	×	×	×		
5300·75	×		×		
5324·32 *	?	o	×		
5328·28	?		o		

* denotes that the line was measured on one plate only.
B denotes hypothetical bright lines.

only plate on which they can be measured, and their wave-lengths may be liable to errors amounting to 0·25 λ. Only one of them seems to be a Sun-spot line.

In their paper, *Astroph. Jour.*, xxiii. 400–405, Hale and Adams give the wave-lengths of lines in the spectrum of *α Orionis* from λ = 5393–5703. Out of 25 lines given by Hale and Adams, all but three can be identified; and three lines appear which are not given by them. The three lines which cannot be identified are 5393·36, 5407·62, 5418·99. The first is a very doubtful spot line, the second is a line much intensified in spots, the third is not intensified in spots. According to Hale and Adams, the first is a strong line in the star spectrum, the second is a fairly strong line, and the third is weak. The absence of the first two lines is remarkable, and suggests the possibility of change in the spectrum of the star.

Of the three lines not given by Hale, 5520·97 is the strongest, though it is only just measurable: the other two are very weak and not measurable; their approximate wave-lengths are 5449 and 5474.

Flutings in the Green Region of the Spectrum of α Orionis.

Measures have been made of the wave-lengths of two absorption flutings sharp on the violet side and fading off towards the red; the heads of these two are at 5166·8 and 5447·1. The first of these flutings is the more marked, possibly on account of the fact that the lines b_4 and b_3 are involved near the head. The second also has a line at its head, whose wave-length is given in the table. According to Professor Fowler, the wave-lengths of the heads of the two strongest titanium flutings are 5167·5 and 5447·0, and their intensities he gives as 10. There seems little doubt then that the two flutings in *α Orionis* are titanium flutings. There are in this region 5 other titanium flutings, whose wave-lengths and intensities are given by Fowler, namely:

5241·0	intensity 5
5308·0	3
5356·6	4
5407·0	1

All of these are to be found in the spectrum of *o Ceti*, but there is no certain indication of their presence in *α Orionis*.

On the Presence of Tin in Stellar Atmospheres. By Joseph Lunt, B.Sc., F.I.C., Assistant at the Royal Observatory, Cape of Good Hope.

During the course of measurements of stellar radial velocities, Mr A. W. Goatcher happened to measure a line at $\pm\lambda$ 4525 on 14 negatives of the spectrum of α Scorpii. He pointed out that the line in question was markedly discordant, giving a velocity which was more than 6 kilometres per second too low (positive) in the mean, when the wave-length of the stellar line was assumed to be λ 4525·285, corresponding to a blend of the two lines in Rowland's tables at λ 4525·110 Int 0, origin unknown, and λ 4525·314 Int 5, due to iron.

On searching for some cause for this pronounced discrepancy, I found that a strong tin line occurred on the ultra-violet side of the iron line.

Exner and Haschek place its wave-length at λ 4525·00 Int 30, and they give another line at λ 4585·80 Int 20, but they record no other lines brighter than intensity 1 in the region covered by the Cape 4 prism spectrograph.

On taking photographs of the spectrum of tin, with electrodes of the pure tin "fuse wire" ordinarily used in electrical circuits, it was found that whilst the λ 4525·00 line came out strongly, the λ 4585·80 line was entirely absent, and in fact no other line appeared except a faint line at λ 4227 due to a trace of calcium.

This was due to the fact that self-induction was used in the secondary circuit of the 18-inch spark induction coil, as is usual in producing the iron spark as a source of the comparison spectrum for our stellar spectra.

On eliminating the self-induction, the λ 4585·80 line came out much more strongly than the λ 4525·00 line, differing in this respect from Exner and Haschek's relative intensities; but with the exception of air lines no other lines of importance appeared.

The line of lower refrangibility is, in fact, the strongest enhanced line of tin recorded by Lockyer, whilst the λ 4525 line is absent from his list of enhanced lines.

It is evident that the λ 4585 line is not likely to occur in the cooler stars such as α Scorpii, and we are reduced to a single line in our 4-prism region in searching for evidence of the presence of tin in these stars.

The tin line from the "fuse wire" was carefully measured and gave the wave-length λ 4525·01, which shows that it is more likely to coincide with Rowland's solar line λ 4525·009 Int 00 than with the line closer to the iron line, viz. λ 4525·110 Int 0.

If we assume that the Sn and Fe lines have the intensities 3 and 4 respectively in α Scorpii instead of 00 and 5 as in the Sun, and wave-lengths λ 4525·009 and λ 4525·314, according to Rowland's solar wave-lengths, we can well represent the stellar line, which as a blend would then have the mean wave-length λ 4525·183.

Photographs of the tin and iron spectra taken on the same plate show the blended lines as a single unresolved broad line with the dispersion employed, just as is shown in the stellar spectra.

We must therefore conclude that, so far as our limited range of spectrum is concerned, a line corresponding to the one and only line of tin which seems likely to occur in the cooler stars is certainly present, and further confirmation of the existence of this metal in stars should be looked for outside the region of spectrum here considered, particularly in the region of longer wave-lengths.

Professors Hale and Adams* have secured with the Snow telescope a spectrum of α Orionis which extends to λ 5703·73. It will be of much interest to ascertain whether the arc line of tin in the yellow region at λ 5631·91 (Kayser and Runge) is shown in their photograph of the spectrum of this star. It occurs in a region of the solar spectrum which is free from strong lines or groups. It is shown as a strong line in Hagenbach and Konen's Atlas, and has also been recorded by Huggins, Thalén and Lecoq de Boisbaudran.

In the Harvard Annals a line is recorded at λ 5632·0 as being stronger in α Orionis than in the Sun, α Aurigæ and α Boötis.

The star line at λ 4525 has been used in radial velocity determinations, once by Frost† in β Leporis and once by Belopolsky‡ in γ Cephei, and in each case the line gives a lower result than any other on the plate. These two stars belong to classes G and K respectively, according to the Harvard classification, and the discordance of the tin-iron line is therefore likely to be less marked than in α Scorpii.

The recorded evidence of the presence of tin in the Sun appears to be very meagre and doubtful.

In "Rowland's table of solar elements,"§ arranged according to number of lines associated with them, tin appears at the end of the list amongst elements showing only one or two coincidences between the lines of their spectra and solar lines.

In Rowland's "Preliminary table of solar spectrum wave-lengths," the only references to Sn seem to be the following three lines :—

Rowland's.			Kayser and Runge (arc).		Exner and Haschek (spark).	
λ	Int.	Origin.	λ	Int.	λ	Int.
3262·409	3	Fe-Sn	3262·44	2	3262·48	30
3330·745	000	Sn?	3330·71	3	3330·83	6
3801·163	000	Sn?	3801·16	3	3801·32	20

These are outside the 4-prism region of the Cape Spectrograph, and no photographs of star spectra are here available to search for these lines in the spectra of the cooler stars.

* *Ap. J.*, vol. xxiii. p. 400.
† *Ap. J.*, vol. xviii. p. 257.
‡ *Ap. J.*, vol. xix. p. 101.
§ *Problems in Astrophysics*, Clerke, p. 27.

Royal Observatory, Cape of Good Hope :
1907 *March* 27.

Note on the Range in Brightness at Maximum of Long-period Variables. By H. H. Turner, D.Sc., F.R.S., Savilian Professor.

1. In an interesting letter dated April 16, commenting upon the paper "On the Classification of Long-period Variable Stars, etc." (*Mon. Not.*, lxvii. p. 332), Mr C. L. Brook draws attention to the possibility of explaining the well-known differences between separate maxima in terms of the hypothesis put forward in the paper. His words are as follows:—

"Not only does the amount of solar disturbance vary greatly at different Sunspot maxima, but also the two hemispheres differ to a considerable extent. . . . Now if the Sun were viewed poleward instead of equatorially, the effect of this would be to further increase the difference that already exists in the intensity of the various maxima: we should either see the whole of the then more disturbed hemisphere or the whole of the less disturbed hemisphere, and not as now half (about) of each. If then the analogy between the Sun and the long-period variables holds in this particular, then the maxima of those variables which we view polewards should differ to a greater extent in intensity than the maxima of those which are viewed equatorially."

2. A rough test of the interesting corollary indicated by Mr Brook is simply made. On pages 351 and 352 of the paper above quoted are two lists of twenty stars from Chandler's Revised 3rd Catalogue which should be (on the hypothesis put forward) stars viewed polewards and stars viewed equatorially. We need only take from Chandler's 3rd Catalogue (*A.J.*, 379) his tabulated range of magnitude at maximum (not repeated in the Revision, *A.J.*, 553). Thus for X Ophiuchi (No. 6682), Chandler gives the magnitude at maximum as 6·8–7·0, or a range of 0·2 magnitude; and in most cases he gives figures of this kind; while the range is often much larger, as for o Ceti (No. 806), where he gives 1·7–5·0, a range of 3·3 magnitudes. In a few cases he gives a single value only, as for RS Libræ (No. 5511), where he sets down 8·2 simply: and in the subjoined table the range has in such cases been set down as 0·0; for though further observations may show an appreciable range, it will no doubt remain small.

3. The following table will be understood without further explanation; means have been taken for each ten stars, and it will be seen that for the extreme groups of ten the difference in range is quite marked, but is *in the opposite sense* to that suggested by Mr Brook.

4. It is not, however, impossible to frame an explanation of this table in terms of the hypothesis put forward. If the variations in the two hemispheres are related by some system of alternation—if, for instance, the total quantity of bright, flocculous matter on the star at maximum were always the same, but distributed for the greater part sometimes in one hemisphere and sometimes in the other—then the view enunciated by Mr Brook is the correct one.

Range at Maximum of the Forty Variables tabulated on pages 351–2.

Large Values of α or "Polar" Stars.			Small Values of α or "Equatorial" Stars.		
α	Star's No.	Range at Max.	α	Star's No.	Range at Max.
+·25	6682	0·2	−·63	1582	0·7
+·22	8591	0·2	−·42	112	3·0
+·19	5511	0·0	−·35	2528	1·2
+·18	5795	1·2	−·34	5504	1·7
+·17	5887	0·5	−·34	4315	0·6
+·16	3637	0·0	−·31	2946	2·3
+·13	715	0·7	−·27	3825	2·2
+·12	107	1·0	−·27	6871	0·0
+·11	7659	0·9	−·27	7045	2·1
+·10	7779	1·8	−·27	243	0·8
	Mean . .	0·65		Mean . .	1·46
+·08	7609	1·6	−·26	5095	0·3
+·08	1717	1·1	−·25	806	3·3
+·07	7577	1·0	−·23	2539	0·7
+·07	7754	1·3	−·22	4492	1·4
+·07	5338	0·9	−·22	3170	1·2
+·05	7428	2·7	−·22	6849	1·5
+·05	5190	1·4	−·20	2100	1·1
+·03	1855	1·3	−·20	5194	0·7
+·03	294	0·0	−·19	434	1·1
+·02	6905	1·0	−·19	976	0·7
	Mean . .	1·23		Mean . .	1·20

Viewing the star equatorially, we should get the combined brightness of both hemispheres, which would be nearly the same; viewing the star polewards, we should get the full effect of the alternation between hemispheres.

5. Suppose, however, that the variations in the two hemispheres are independent of each other: or, if related, related on the principle of sympathy rather than that of alternation. In either case we might have them both bright (at maximum) or both faint (at maximum) together; and the range for equatorial view would not be any smaller than for polar view. But it would not, in the long run, be greater. Within a limited period it might seem to be greater, because one hemisphere might be regular for a limited period: and if the star were seen poleward the maxima

would resemble each other, whereas if seen equatorially the irregularities in the other hemisphere would come in. The problem is very similar to the well-known problem of bimetallism.

6. There is, however, in our problem a factor not represented in that of bimetallism. Increased activity may mean the production of increased clouds of absorbing Corona, and the effect of this on the maximum might be different in different aspects. It would be unprofitable to speculate on the importance of this unknown factor in the present state of our knowledge. It seems better simply to call attention to the above fact. It is, on the face of it, unfavourable to the hypothesis put forward: but a direct contradiction is more satisfactory than a fact which goes neither one way nor the other, since there is always the possibility that a simple inversion of some kind has been overlooked.

Elements of the Variable Star RV Andromedæ. R.A. $= 2^h\ 4^m\ 34^s$, Decl. $= +48°\ 27'{\cdot}6$ (1900). By A. Stanley Williams.

A period of 182 days was suggested for this variable star in 1904 by the writer, from a comparison of a single visually-observed minimum in January of that year with some earlier photographic observations.* From the observations made here last year, combined with two maxima and two minima observed by Professor A. A. Nijland at Utrecht,† the following much more satisfactory elements of variation have been derived:

$$\text{Maximum} = \left\{\begin{array}{l}\text{1904 April 26}\\ \text{J.D. 2416597}\end{array}\right\} + 170^d{\cdot}2\ \text{E}$$

the interval M – *m* being 82 days.

The above period was derived from five observations of four maxima and seven observations of six minima, the periods resulting from the maxima and minima being respectively 172·2 days and 166·2 days. The observations of the minima were more numerous than those of the maxima, and extended over a longer interval of time; but the maxima were more sharply defined than the minima, and the observations also more accordant, so that double weight was given to the determination from the maxima.

Quite recently the dates of two additional maxima and two additional minima have been published by Nijland;‡ and those of two maxima and a minimum by Mr F. H. Seares, from observations made by Mr E. S. Haynes at the Laws Observatory.§ Table I. below contains the particulars of all the observed maxima, together with the computed dates according to the foregoing elements, and the differences O – C. Table II. gives the corre-

* *A.N.*, No. 3944. † *A.N.*, No. 4116. ‡ *A.N.*, No. 4164.

§ *Bulletin No.* 10 *of the Laws Observatory*, p. 147. Seares derived a period of 169 days for the star.

sponding particulars for the minima. In both tables the observations published subsequently to the derivation of the elements are marked with an asterisk.

TABLE I.

Observed and Computed Maxima.

E	Obs. Max.	Comp. Max.	O − C.	Obs. Mag.	Observer.
			d		
1	2416767	2416767·2	− 0·2	8·3	Williams
2	6934	6937·4	− 3·4	9·0	Nijland
3	7103	7107·6	− 4·6	8·0	Williams
3	7105	7107·6	− 2·6	8·9	Nijland
4	7289*	7277·8	+11·2	8·9	Nijland
5	7445*	7448·0	− 3·0	...	Haynes
5	7453*	7448·0	+ 5·0	9·1	Nijland
5	7455	7448·0	+ 7·0	8·2	Williams
6	7609*	7618·2	− 9·2	...	Haynes

TABLE II.

Observed and Computed Minima.

E	Obs. Min.	Comp. Min.	O − C.	Obs. Mag.	Observer.
			d		
0	2416509	2416515·0	− 6·0	10·1	Williams
1	6694	6685·2	+ 8·8	10·2	Williams
2	6875	6855·4	+19·6	10·4	Williams
3	7045	7025·6	+19·4	11·1	Nijland
4	7174	7195·8	−21·8	10·8	Nijland
4	7177	7195·8	−18·8	10·3	Williams
5	7341*	7366·0	−25·0	10·8	Nijland*
6	7514*	7536·2	−22·2	...	Haynes
6	7518*	7536·2	−18·2	11·3	Nijland
6	7518	7536·2	−18·2	10·6	Williams

The residuals in the fourth columns of the preceding tables are somewhat large, particularly those in Table II., but having regard to all the circumstances, it does not seem that any important correction to the foregoing elements of variation is indicated by the subsequently published observations. One somewhat interesting feature is, however, pointed out. The progressive nature of the change in the residuals of Table II. seems to show that the interval from minimum to maximum is not constant in the case of this star, but varies to a very appreciable extent from time to

* Nijland also observed a secondary minimum on J.D. 2417405 (O − C = +61^d), with an intervening secondary maximum, about half a magnitude brighter.

time. In other words, this would mean that the form of the light curve is, doubtless, within limits, in a state of more or less constant change. It is possible that the change here referred to may be due, in part at least, to the existence of a periodical irregularity in the length of the period of variation, but the observations hardly favour this, though they have not yet been continued long enough to settle the question. It should be noted that the largest and most important change indicated by the residuals of Table II. is supported by the accordant observations of two different observers, so that there cannot be much doubt as to its reality.

With regard to the magnitudes in the penultimate columns of the tables, it should be remarked that the systematic difference between Nijland and the writer is probably due to a difference of scale. The comparison stars made use of at Utrecht were different from those used here. In particular, Nijland used the star BD. $+47^{\circ}\cdot573$ ($8^{m}\cdot3$) when the variable was bright, whereas at such times the star BD. $+47^{\circ}\cdot571$ ($8^{m}\cdot2$) was employed by the writer. At the Laws Observatory the comparisons were made with another different star, BD. $+48^{\circ}\cdot612$ ($9^{m}\cdot3$), Seares stating that Haynes's observations show a range of $2^{m}\cdot8$, with a maximum $1^{m}\cdot0$ brighter than the last mentioned star. Probably we shall not be far wrong if we adopt a range of variation from $8\frac{1}{2}$ mag. to 11 mag.

The light curve of this variable, according to the observations made here, is very acute at the time of maximum, and the decline very quick. But after the minimum there is sometimes at first only a slight increase in brightness, followed by a long standstill; or even by a secondary minimum, with an intervening secondary maximum, as observed by Nijland in 1906.

The spectrum of RV Andromedæ appears to be peculiar. Thus the Rev. T. E. Espin recorded it as III !!! in 1893,* whilst Mr F. Krüger described it as being III !! (?) in 1902, the bands appearing broad and deep, and the blue and violet wanting.†

On the Variable Star RX Andromedæ. R.A. $= 0^{h}\ 58^{m}\ 55^{s}$, Decl. $= +40^{\circ}\ 46'\cdot2$ (1900). By A. Stanley Williams.

The observations of this variable star, which were made here in the year 1905, have already been published in full in the *Monthly Notices*, vol. lxvi. pp. 336, 337. These showed that the star in that year underwent many sudden and irregular variations in brightness, of such a character that, taken in connection with the earlier observations, it seemed as though the variable should be regarded as belonging to the highly-remarkable type of *U Geminorum*.

Between July 24 and December 21 of last year, 48 observations of the brightness of the star were made on 45 nights, but its

* *A.N.*, No. 3231. † *A.N.*, No. 3991.

history during the period of time covered by the observations proved of a most uneventful and quiet character. With the exception of the few occasions hereafter more particularly referred to, when a comparatively slight increase in brightness was manifested, the variable remained almost unchanged at about 9 of the provisional light scale,* corresponding to 11·6 mag., that is, at about its minimum brightness.

It seems unnecessary, under the circumstances, to set out all these observations in full, and I therefore give below, in Table I., simply the dates when the star was observed faint or near its minimum brightness.

TABLE I.—*Dates when RX Andromedæ was observed faint.*

1906 Aug. 19, 21, 22, 25, 27, 28, 29, 30, 31; Sept. 1, 6, 9, 10, 15, 16, 19, 20, 22, 24, 25, 26, 27, 29; Oct. 23, 25, 27, 28; Nov. 11, 15, 17, 19, 22; Dec. 21.

TABLE II.

Observations made when RX Andromedæ was bright.

Date.	G.M.T.	Observations.	Sky.	Brightness.
1906 July 24	13h 10m	d 3 v 8 e, c 3 v	IV.	19·5
25	13 34	d 7 v 5 e, c 8 v	IV.	17·6
27	13 25	d 7 v 3 e, c 9 v	IV.	16·7
Aug. 16	11 46	d 8 v 1 e	IV.	15·6
Sept. 11	11 33	d 3 v 4 e, c 2 v†	IV.	18·8
,,	13 47	d 2 v 5 e, c 2 v	IV. ☽	19·4
12	16 03	d 5 v 0 e, c 6 v	IV. ☽	15·3
Oct. 5	15 47	d 5 v 0 e‡	III. ☽ ☽ ☽	15·2
6	12 57	d 4 v 5 e	III. ☽ ☽	18·1
10	9 38	v 2 d, v 9 e, v 2 c	III.	23·4
13	10 07	d 6 v 0 e	III.	14·9
14	8 46	d 3 v 3 e	IV.	17·8
Dec. 21	10 58	d 5 v 2 e	III.	16·6

The last column gives the brightness according to the provisional light scale. It will be seen that none of the observations of Table II. make the star very bright, and although it is possible that in one or two cases the actual maximum may have happened at a time when there are no observations available, yet this certainly could not have been the case as regards the temporary increase in brightness that occurred on September 11 and 12, since there are observations both before and after these dates showing that the

* *Monthly Notices*, vol. lxvi. p. 336, Table I.
† Light of variable very unsteady.
‡ Sky bright and stars faint but variable, certainly fainter than *d*.

star was then faint. It would seem, therefore, that we are justified in concluding that during the latter half of 1906 the variable remained persistently faint, and nearly at its minimum brightness, during the greater part of the time, and only underwent occasional and comparatively feeble augmentations of brightness, or abortive maxima. The *U Geminorum* type of variation of the star is confirmed by the recent observations, though there seem to be minor differences between all the stars of this class. The range of variation of *RX Andromedæ* is less than that of *U Geminorum*, but the former appears to be more irregular even in its light changes than the latter.

New Double Stars. By the Rev. T. E. Espin, M.A.

The following pairs have been found with the 17¼ in. Reflector during the spring. The weather has been persistently unfavourable since the end of March, and few measures have been obtained.

No.	B.D.	R.A. 1900 h m	Decl. ° ′	P. °	D. ″	Mags.	Nights.	Date. 1900.	
404	+56°,114	0 37·2	+57 5	68·1	2·90	9·3 12·5	1	7·063	
405	+57,171	49·8	57 15	114·8	4·17	9·0 9·0	2	7·043	
406	+56,153	50·9	56 52	149·1	3·67	9·2 9·5	2	7·073	
407				90·1	3·55	9·4 11·0	2	7·073	
408	+57,251	1 13·3	57 45	160·1	2·70	9·3 11·2	2	7·073	
	408 407			345·2	89·98		3	7·094	
409	+55,506	2 0·6	55 39	108·7	1·95	9·5 11·0	2	7·106	
410	+34,769	3 50·2	34 10	102·8	2·88	9·4 9·8	3	7·073	
411	+33,752	52·1	33 40	46·2	4·47	9·2 9·3	2	7·073	
412	+32,876	4 57·4	32 14	267·6	4·77	8·5 12·0	3	7·108	
413	+32,880	58·6	32 13	5·4	5·93	9·0 13·5	3	7·108	
414	+33,975	5 6·3	33 25	183·8	2·38	9·0 9·1	3	7·129	
415	+32,1109	44·8	32 6	16·1	15·00	6·9 12·5	2	7·089	
416	+31,1219	6 4·1	31 33	197·7	7·65	8·9 11·5	2	7·166	
417	+33,1265	4·2	33 1	330·0	13·55	8·0 12·5	2	7·206	AB
				237·0	14·50	14·5	2	7·206	AC
				275·5	37·55	11·0	2	7·206	AD
418	+35,1600	7 16·8	35 12	15·7	13·20	8·1 13·3	3	7·166	
419	+34,1641	28·2	33 56	125 6	3·58	10·0 10·7	3	7·073	BC
				44·0	58·30	A = 9·4	2	7·065	AB
420	+29,1597	39·5	28 59	273·4	2·65	12·5 13·0	1	7·246	BC
				118·1	60·79	A = 8·7	1	7·246	AB
421	+29,1632	46·1	28 57	217·8	6·20	9·0 12·0	2	7·243	

No.	B.D.	R.A. 1900 h m	Decl. ° ′	P. °	D. ″	Mags.		Nights.	Date. 1900.	
422	+29,1645	49·2	+29 25	181·3	14·86	8·1	10·5	1	7·246	
423	+34,1741	59·4	34 53	298·9	3·21	8·5	10·4	3	7·073	
424	+35,1756	8 0·6	35 15	73·7	5·77	9·1	10·5	3	7·073	
425	+25,1854	2·3	25 50	257·2	4·52	8·1	12·5	3	7·223	
426	+28,1597	18·8	28 47	284·0	4·85	9·0	9·2	1	7·246	
427	+29,1821	39·9	29 22	72·4	2·50	9·0	10·2	2	7·183	
428	+28,1774	9 30·7	28 47	20·0	12·55	8·6	9·1	3	7·223	
429	+31,2017	32·6	31 18	10·8	3·82	9·5	9·5	2	7·239	
430	+29,1992	10 0·0	29 47	170·8	1·45	9·4	9·7	4	7 230	
431	+27,1852	2·5	27 17	198·7	3·95	10·5	10·6	2	7·282	BC
				347·3	41·87	A = 8·0		1	7·246	AC
				351·6	44·20			1	7·317	AB
432	+33,1988	20·1	33 8	160·4	2·52	9·3	9·6	2	7·232	
433	+30,2087	51·7	30 16	220·0	5·75	9·3	10·5	2	7·282	
434	+35,2233	11 13·4	35 32	247·8	7·76	9·3	14·5	2	7·216	
435	+29,2207	39·6	28 59	347·2	6·80	9·2	9·4	2	7·227	
436	+30,2277	12 23·5	30 26	316·5	1·85	9·2	9·2	2	7·269	
437	+30,2281	24·5	30 4	222·0	2·50	8·8	9·4	2	7·242	BC
				2·0	69·63	A = 8·5		1	7·246	AB
438	+26,2382	38·0	26 27	281·0	5·00	8·2	13·0	2	7·232	
439	+28,2155	46·6	27 46	63·8	1·83	8·9	9·4	3	7·237	
440	+29,2387	13 12·7	29 12	361·5	2·55	9·5	9·5	1	7·301	
441	+28,2211	13·0	28 39	77·5	4·80	8·6	13·2	2	7·238	
442	+28,2251	37·9	28 43	243·2	7·45	8·4	13·0	2	7·234	

Notes.

405.—This pair was found in 1892. The only other measures are mine:

1892·805, P. 116°·6 D. 4″·86 2 nights.

407, 408.—Two pairs a little N of ϕ Cassiopeiæ.

409.—Measures discordant.

417.—The *comes* C is very difficult and the measures are discordant.

Erratum in Pogson's Observations of U Geminorum.

Vol. lxvii. page 129, footnote:—The inverted commas should come at the end of the footnote, and not before the last sentence. The whole note is Pogson's. But Mr C. L. Brook points out that his suspicions are probably unnecessary. "Knott makes U bright on January 4 and 6, but fading: on January 17 he makes it $<13·3$. Baxendell also makes it bright on January 4–6 and <13 on January 20. Pogson's observation falls in with these, and the rapid descent in 5 days from 10·2 to $<12·8$ is not unusual: see April 12 to 17, 10·3 to 13·8."—(Letter of April 16 to H. H. T.)